

Process motifs for covariance and entropy

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Outline

1. Motifs in networks
2. Pipeline
3. Motifs for covariance
4. Motifs for entropy
5. Conclusions

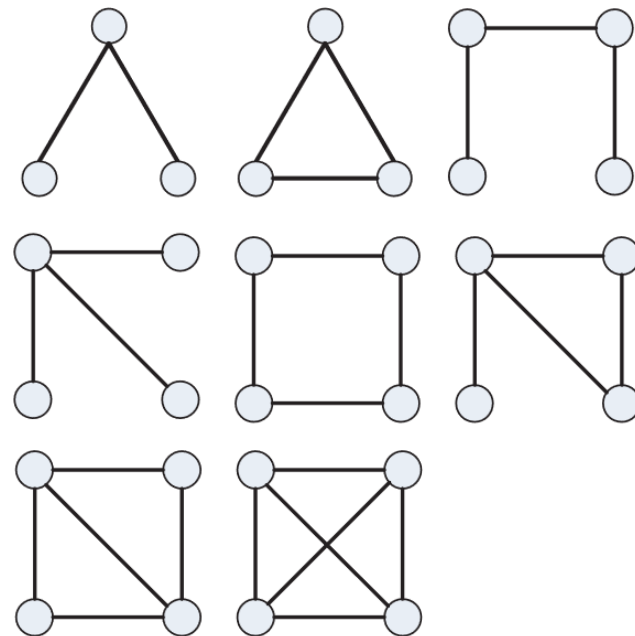
Preprint!!!

<https://arxiv.org/abs/2007.07447>

Motifs in networks

- What is a structural motif?
 - A small, connected subgraph that is **important** for a network's **function**

- What can you do with structural motifs?

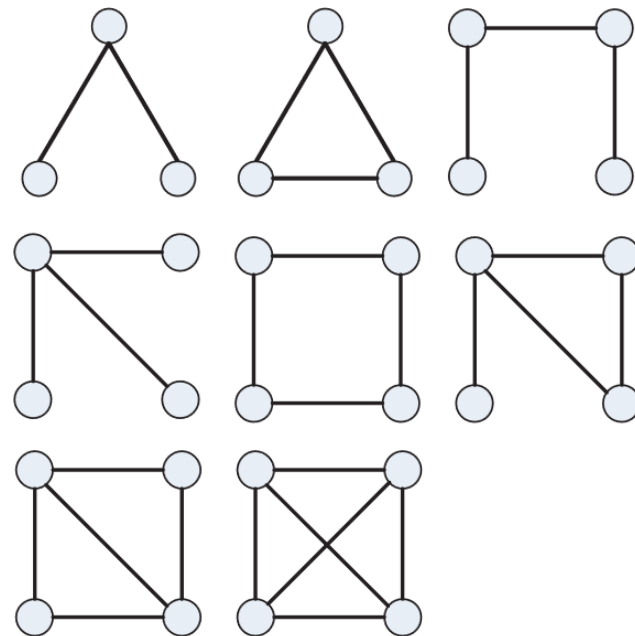


[Wang et al. 2014](#)

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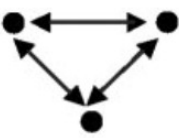
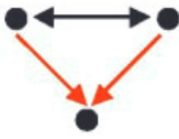
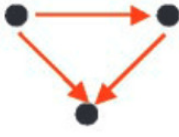
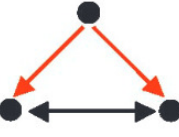
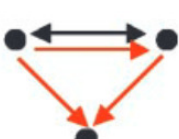


[Wang et al. 2014](#)

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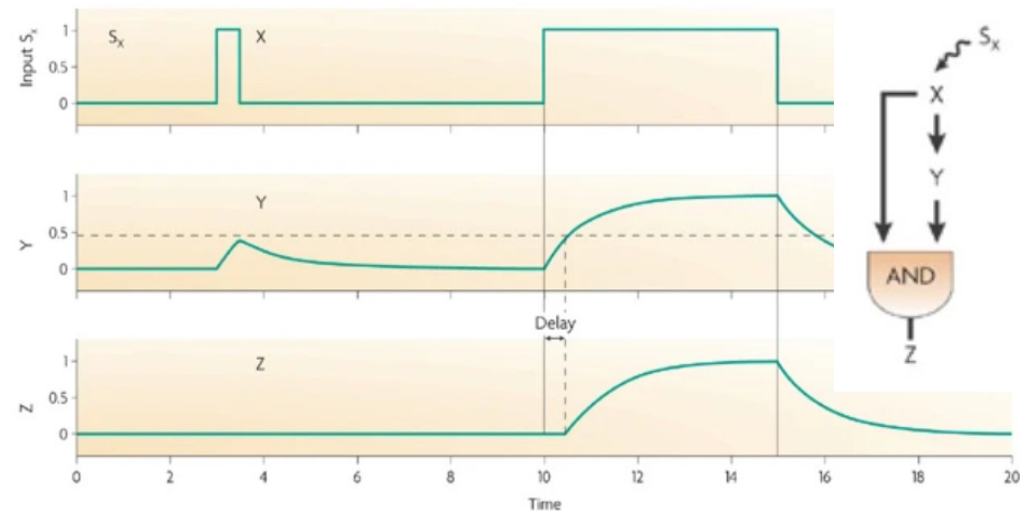
Illustration [†]	<i>N</i> real	<i>N</i> rand ± SD	<i>z</i> score
	1,293	14 ± 3.8	332.7
	243	2.4 ± 2.1	115.9
	83	26 ± 6	9.5
	66	2 ± 1.4	46.5
	46	2.7 ± 1.6	26.3

[Yeager-Lotem et al. 2004](#)

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- What can you do with structural motifs?
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 - Simulate dynamics on **isolated** structural motifs



[Alon 2007](#)

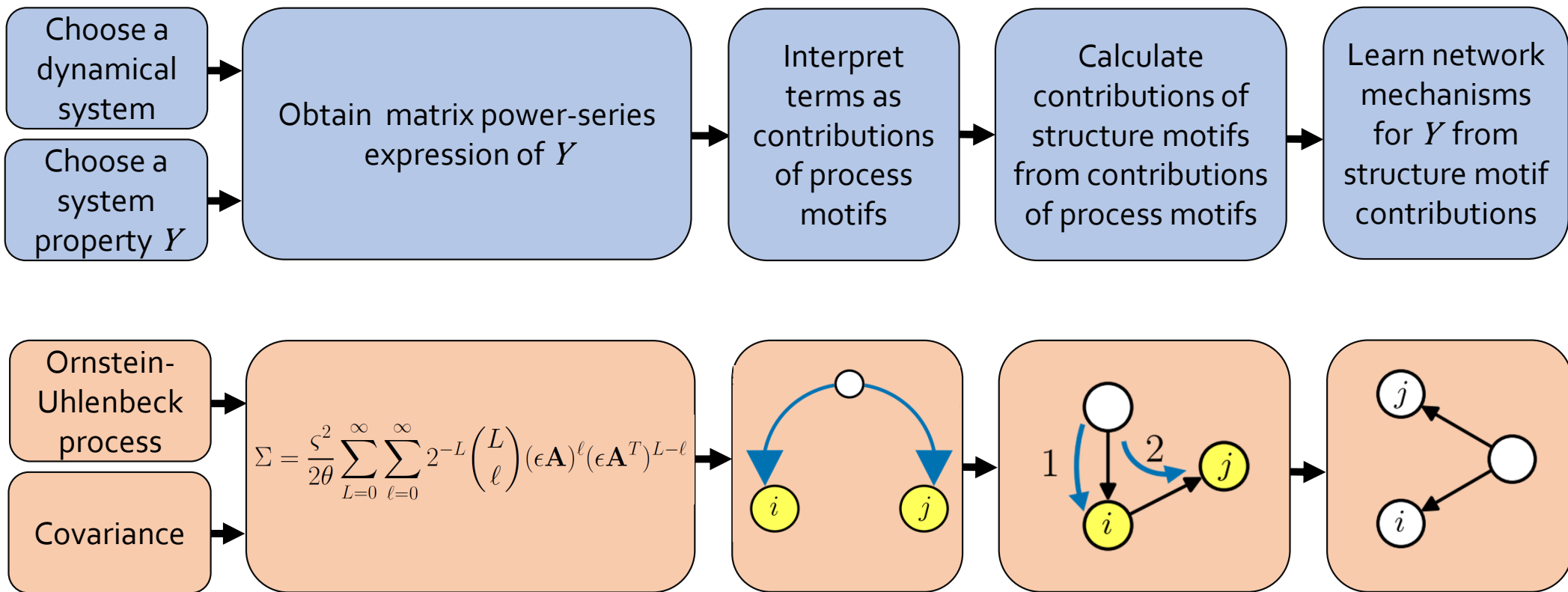
Motifs in networks

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How can we identify motifs that are important for emergent properties of networks?

Pipeline



Choosing a dynamical system

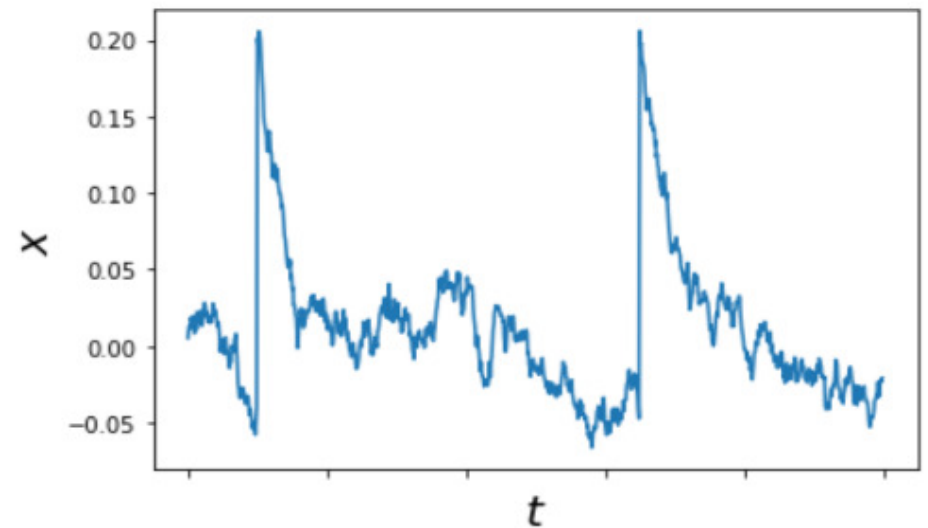
Ornstein-Uhlenbeck process

Simple stochastic differential equation

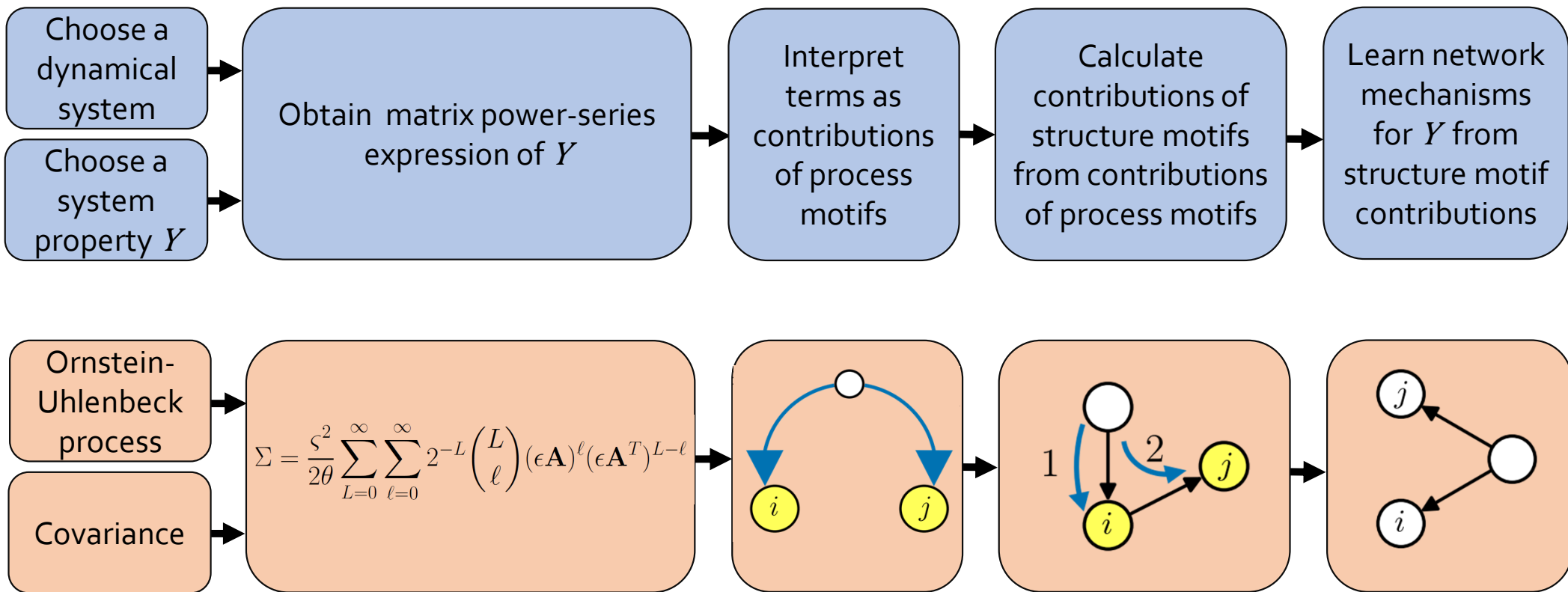
Popular in neuroscience, econometrics, etc.

Linear-response approximation of IF model

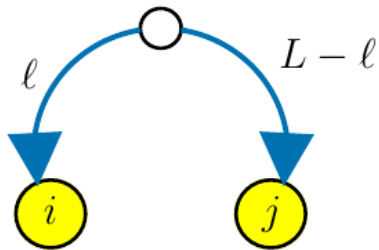
$$d\mathbf{x}_{t+dt} = \theta(\epsilon\mathbf{A} - \mathbf{I})\mathbf{x}_t dt + \varsigma dW_t$$



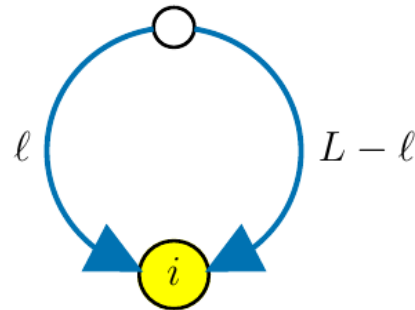
Pipeline



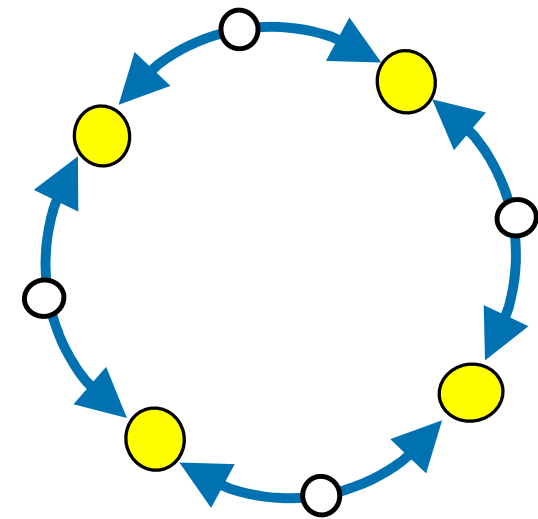
System properties and process motifs



Process motifs
for covariance

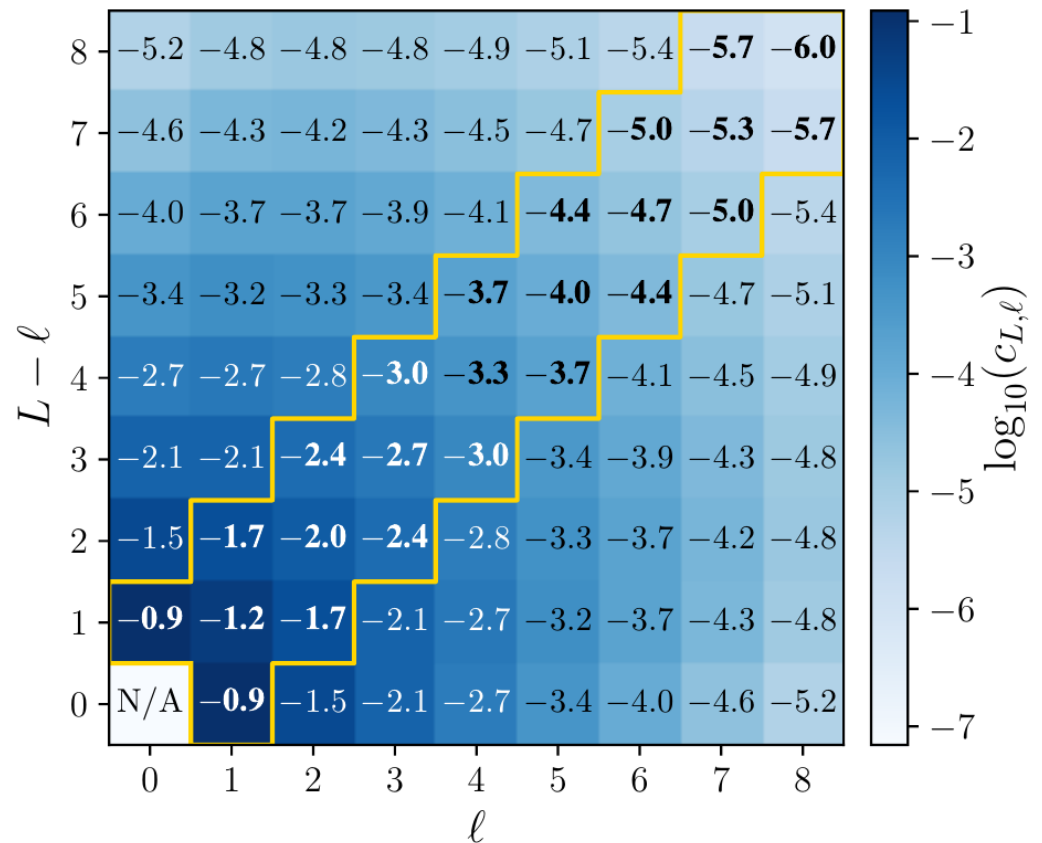
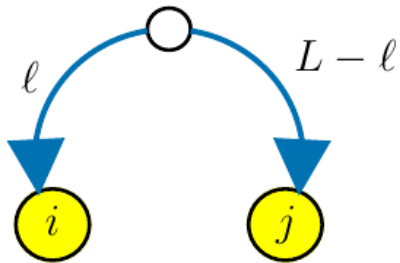


Process motifs
for variance

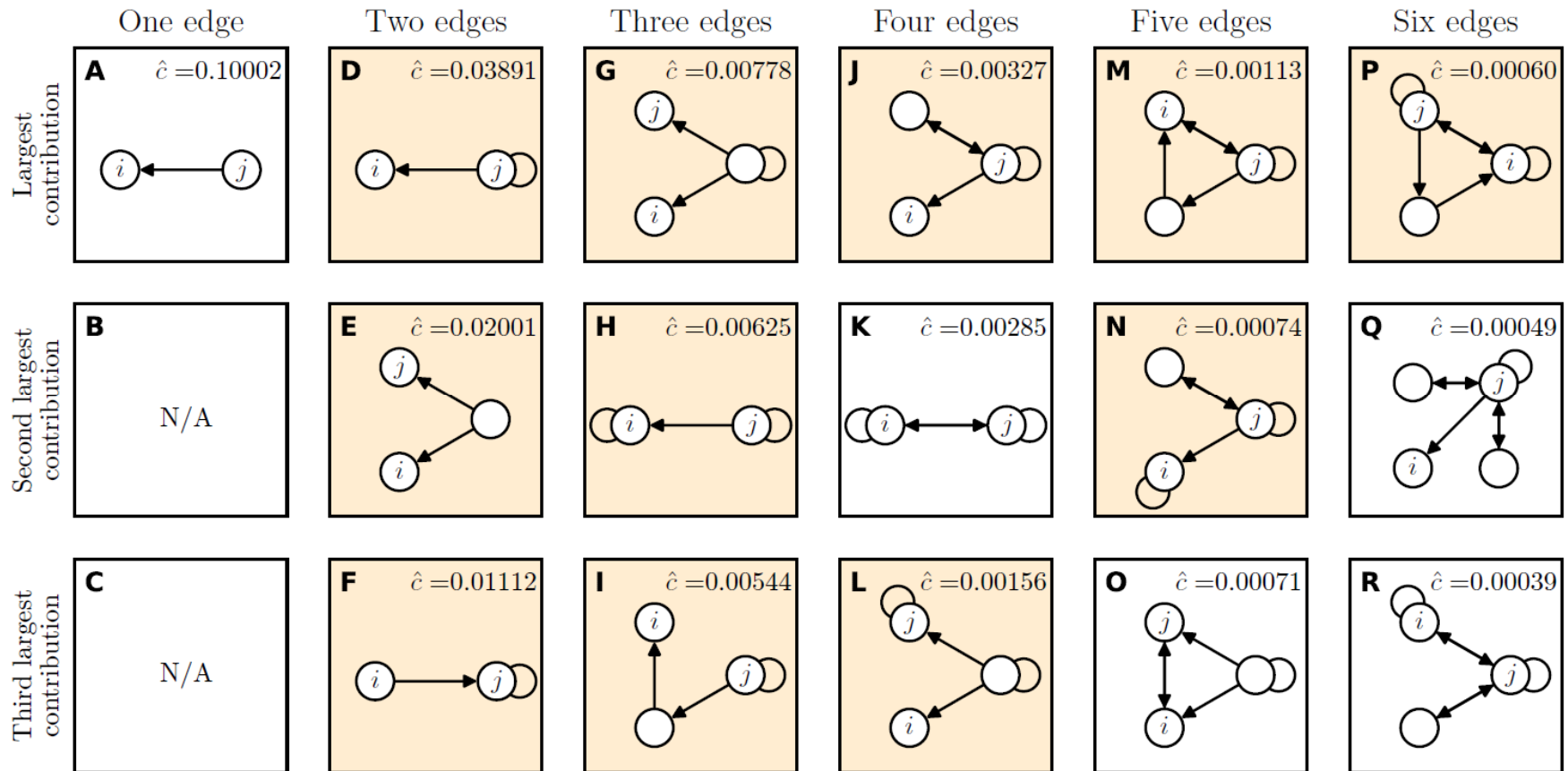


Process motifs
for entropy

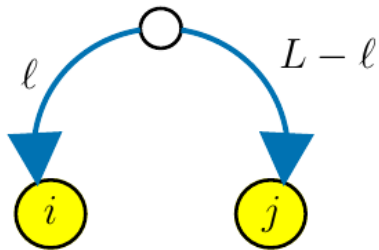
Process motifs for covariance



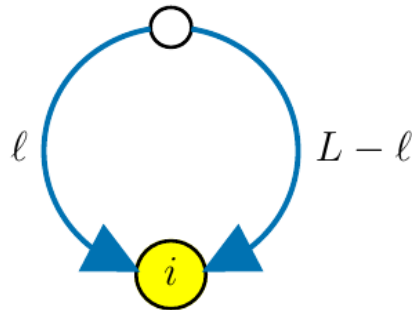
Structure motifs for covariance



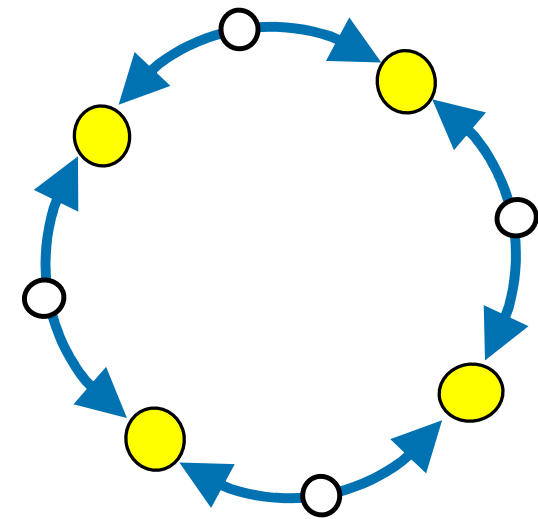
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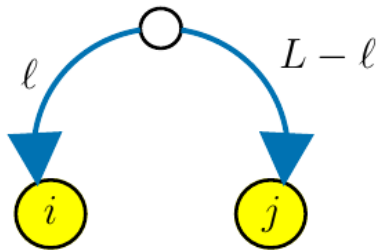


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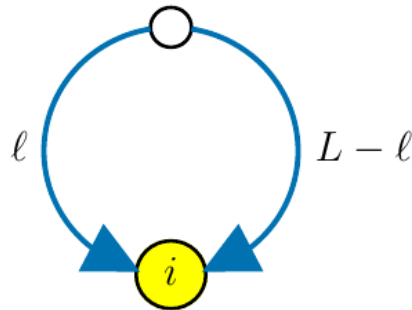


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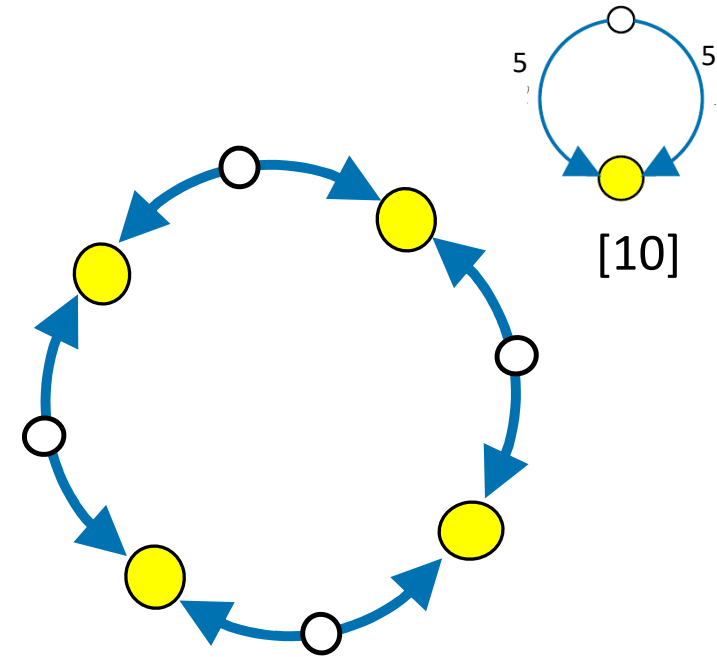
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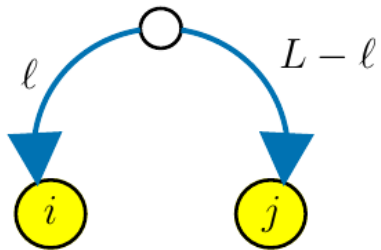


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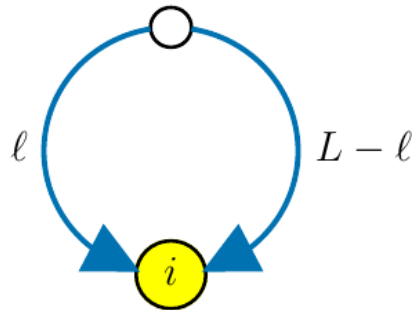


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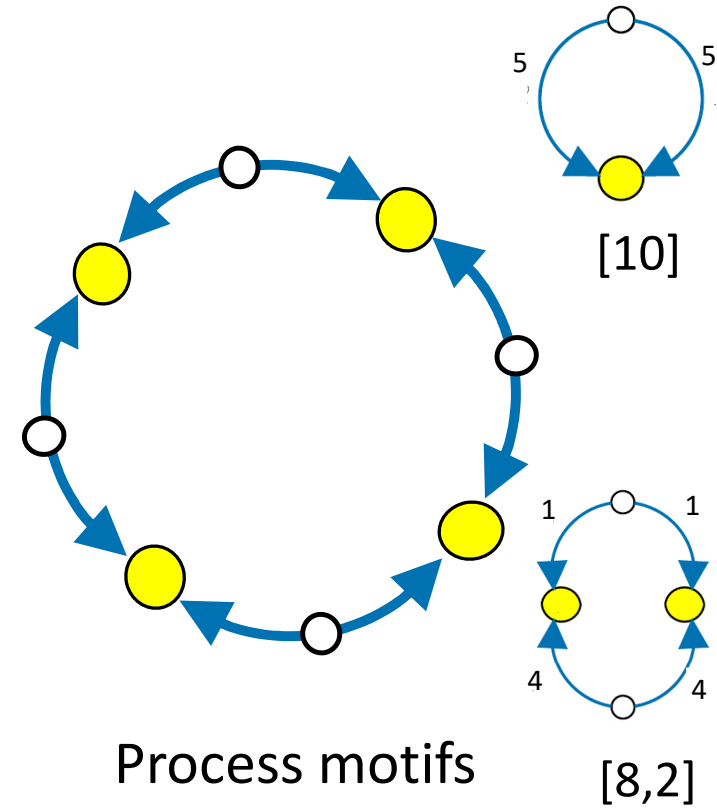
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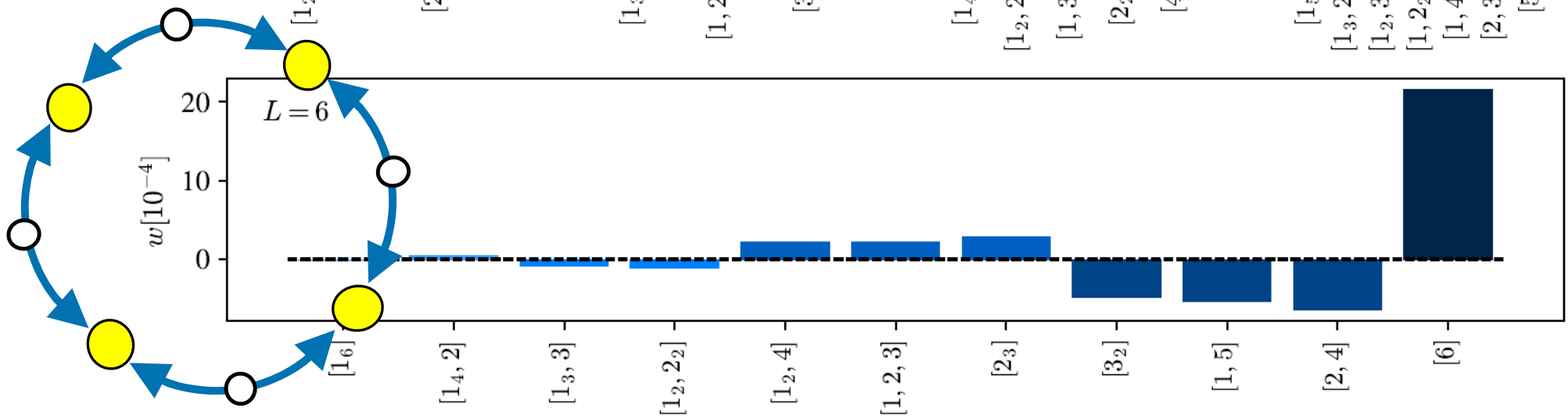
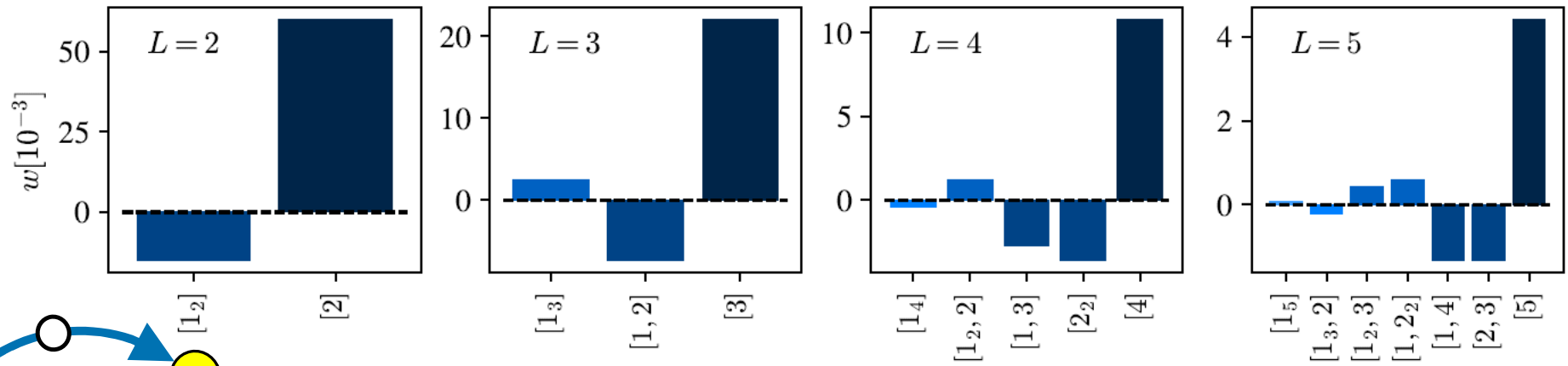


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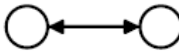
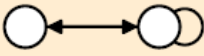
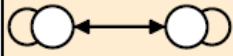
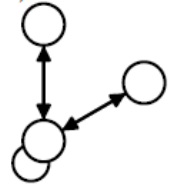
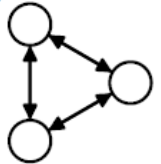

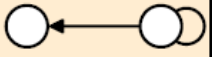

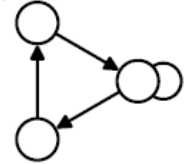
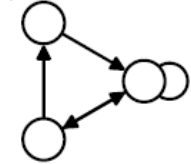
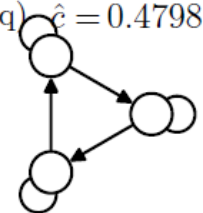
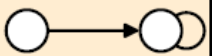
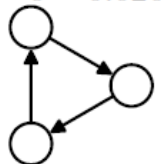
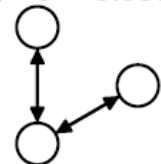
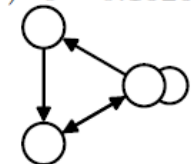
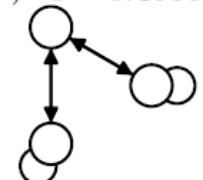


Process motifs
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Process motifs for entropy



Structure motifs for entropy

	One edge	Two edges	Three edges	Four edges	Five edges	Six edges
Largest contribution	(a) $\hat{c} = 0.3367$ 	(d) $\hat{c} = 0.0790$ 	(g) $\hat{c} = 0.1391$ 	(j) $\hat{c} = 0.7179$ 	(m) $\hat{c} = 0.7036$ 	(p) $\hat{c} = 0.5838$ 
Second-largest contribution	(b) $\hat{c} = 0.0291$ 	(e) $\hat{c} = 0.0209$ 	(h) $\hat{c} = 0.0328$ 	(k) $\hat{c} = 0.0363$ 	(n) $\hat{c} = 0.1324$ 	(q) $\hat{c} = 0.4798$ 
Third-largest contribution	(c) N/A	(f) $\hat{c} = 0.0209$ 	(i) $\hat{c} = 0.0202$ 	(l) $\hat{c} = 0.0332$ 	(o) $\hat{c} = 0.1323$ 	(r) $\hat{c} = 0.4668$ 

Conclusions

- A **modeller's perspective on the role of motifs** in networks
- **Flexible, modular framework** for different dynamics and system properties
- **Mechanistic connections** between structural motifs and their contribution to emergent properties of processes on networks
- **Microlevel understanding** of maximum-entropy network structures