Dynamics on networks

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Outline

1. Introduction
2. Motifs for processes on networks
3. Network robustness and system design
4. Conclusions and outlook

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Introduction

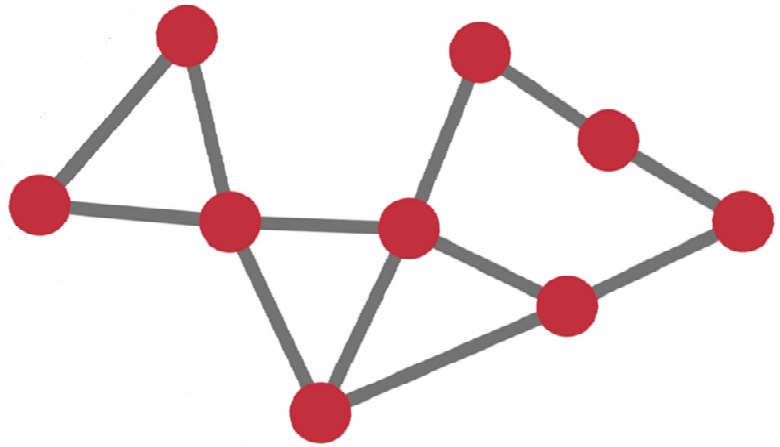
Dynamics on networks

Networks

Dynamics

Dynamics on networks

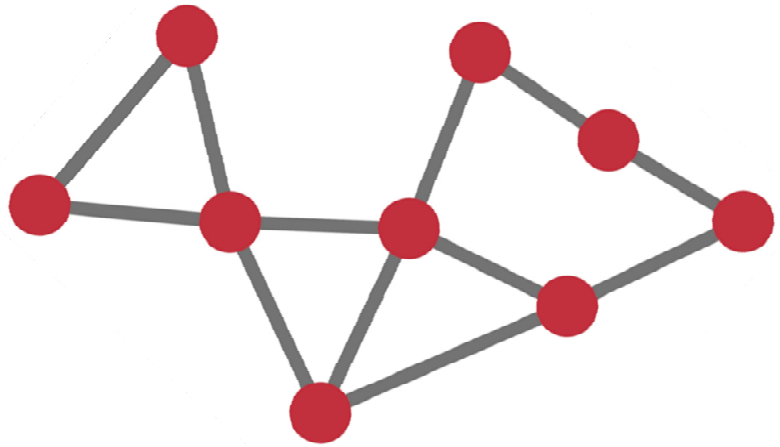
Networks



Dynamics

Dynamics on networks

Networks

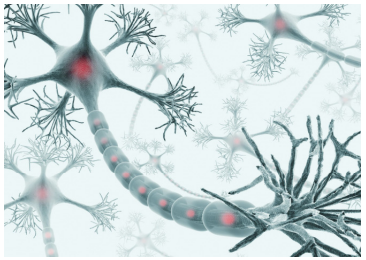


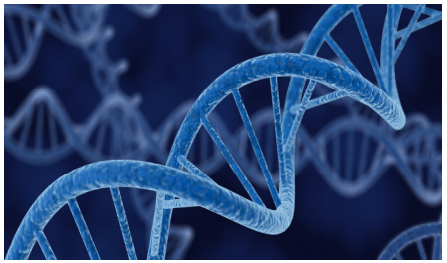
Dynamics

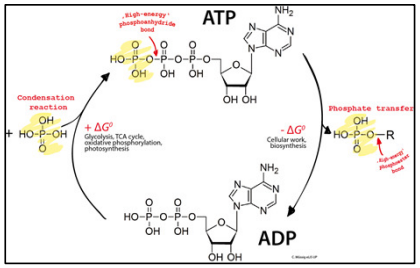
coupled dynamical system

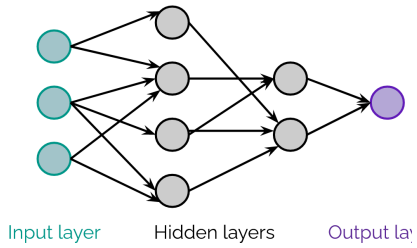
$$\frac{d\mathbf{x}_t}{dt} = \mathbf{F}(\mathbf{A})\mathbf{x}_t$$

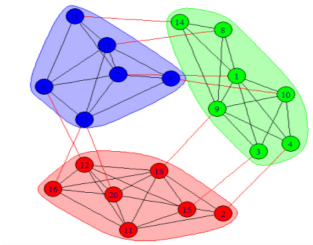
Applications











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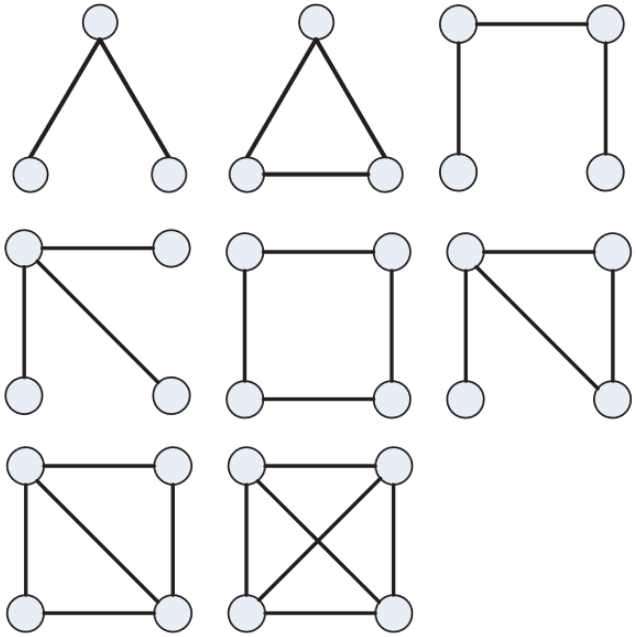
Motifs for processes on networks

Motifs in networks

- What is a structural motif?
  - A small, connected subgraph that is **important** for a network's **function**

- What can you do with structural motifs?



Wang et al. 2014

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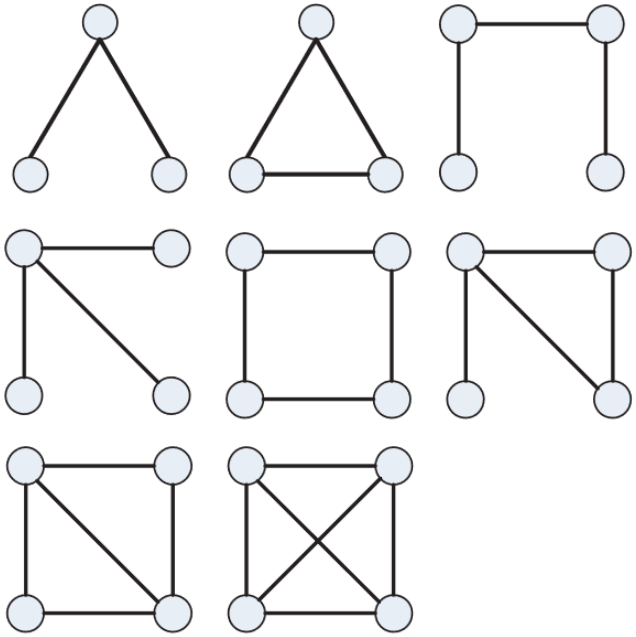

Wang et al. 2014

Motifs in networks

- What is a structural motif?
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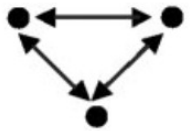
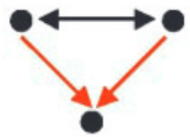
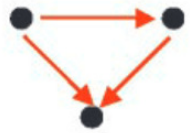
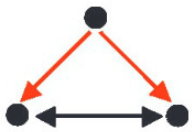
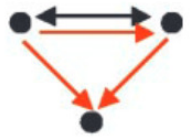
- What can you do with structural motifs?
  - Count them

Illustration†	*N* real	*N* rand ± SD	*z* score
	1,293	14 ± 3.8	332.7
	243	2.4 ± 2.1	115.9
	83	26 ± 6	9.5
	66	2 ± 1.4	46.5
	46	2.7 ± 1.6	26.3

Yeger-Lotem et al. 2004

Motifs in networks

- What is a structural motif?
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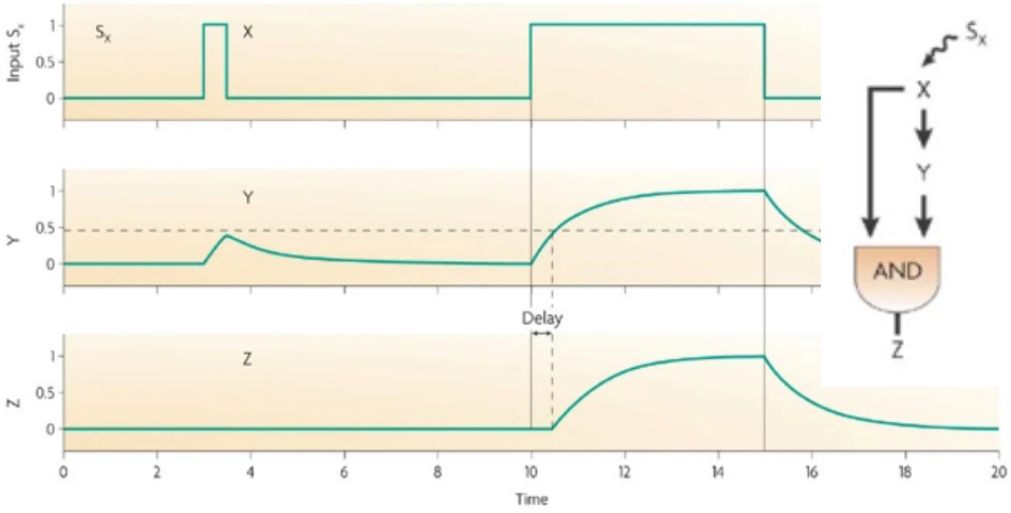
- What can you do with structural motifs?
  - Count them
  - Simulate dynamics on **isolated** structural motifs



Alon 2007

Motifs in networks

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  - Simulate dynamics on **isolated** structural motifs

How can we identify motifs that are important for emergent properties of networks?

Pipeline

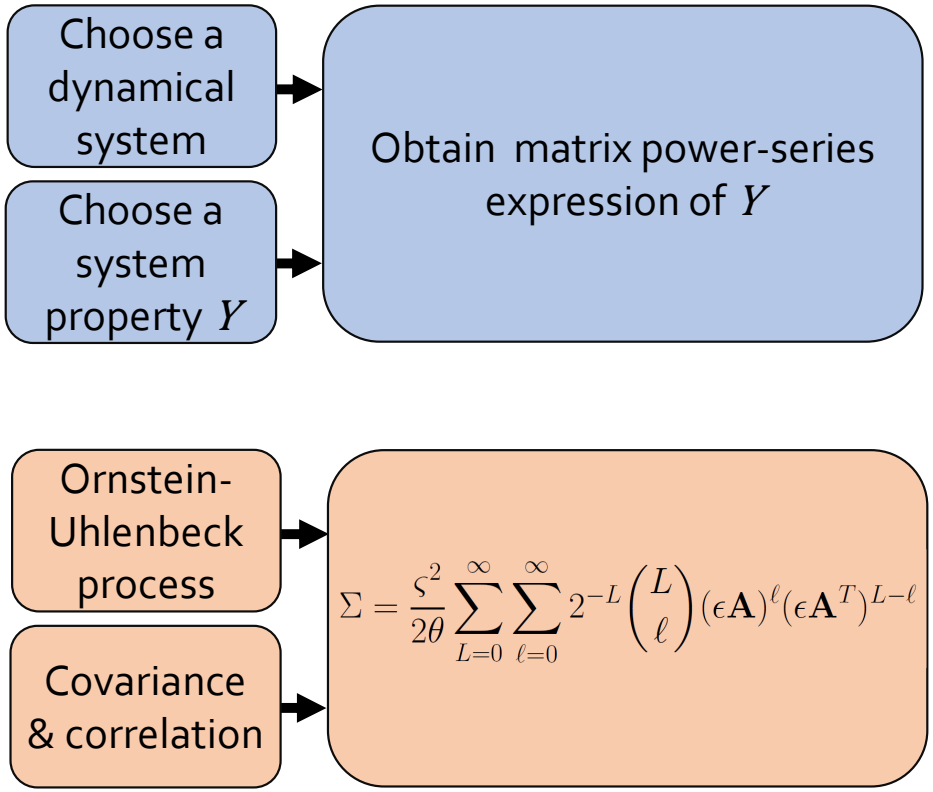
Choose a dynamical system

Choose a system property Y

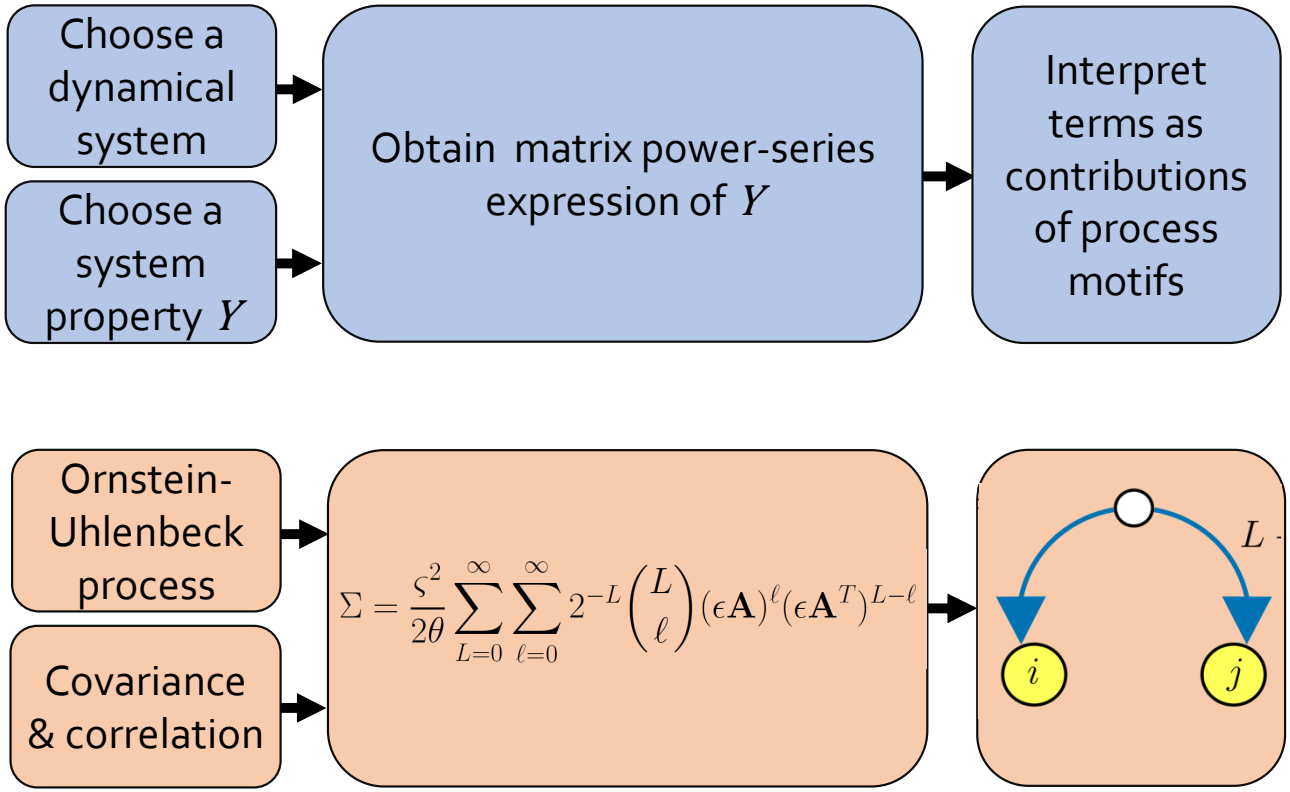
Ornstein-Uhlenbeck process

Covariance & correlation

Pipeline


Choose a dynamical system
Choose a system property Y
Obtain matrix power-series expression of Y
Ornstein-Uhlenbeck process
Covariance & correlation
$\Sigma = \frac{\varsigma^2}{2\theta} \sum_{L=0}^{\infty} \sum_{\ell=0}^{\infty} 2^{-L} \binom{L}{\ell} (\epsilon \mathbf{A})^{\ell} (\epsilon \mathbf{A}^T)^{L-\ell}$

Pipeline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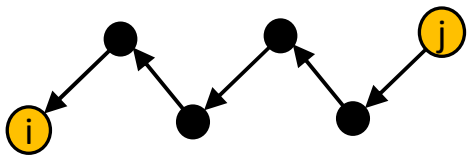
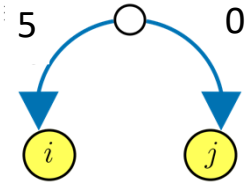
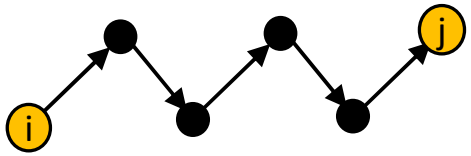
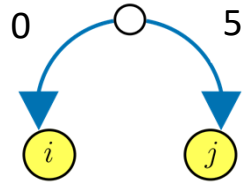
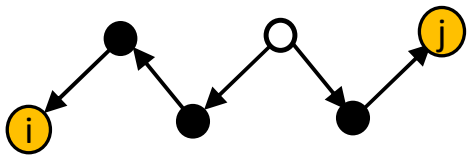
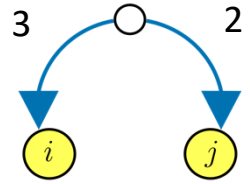
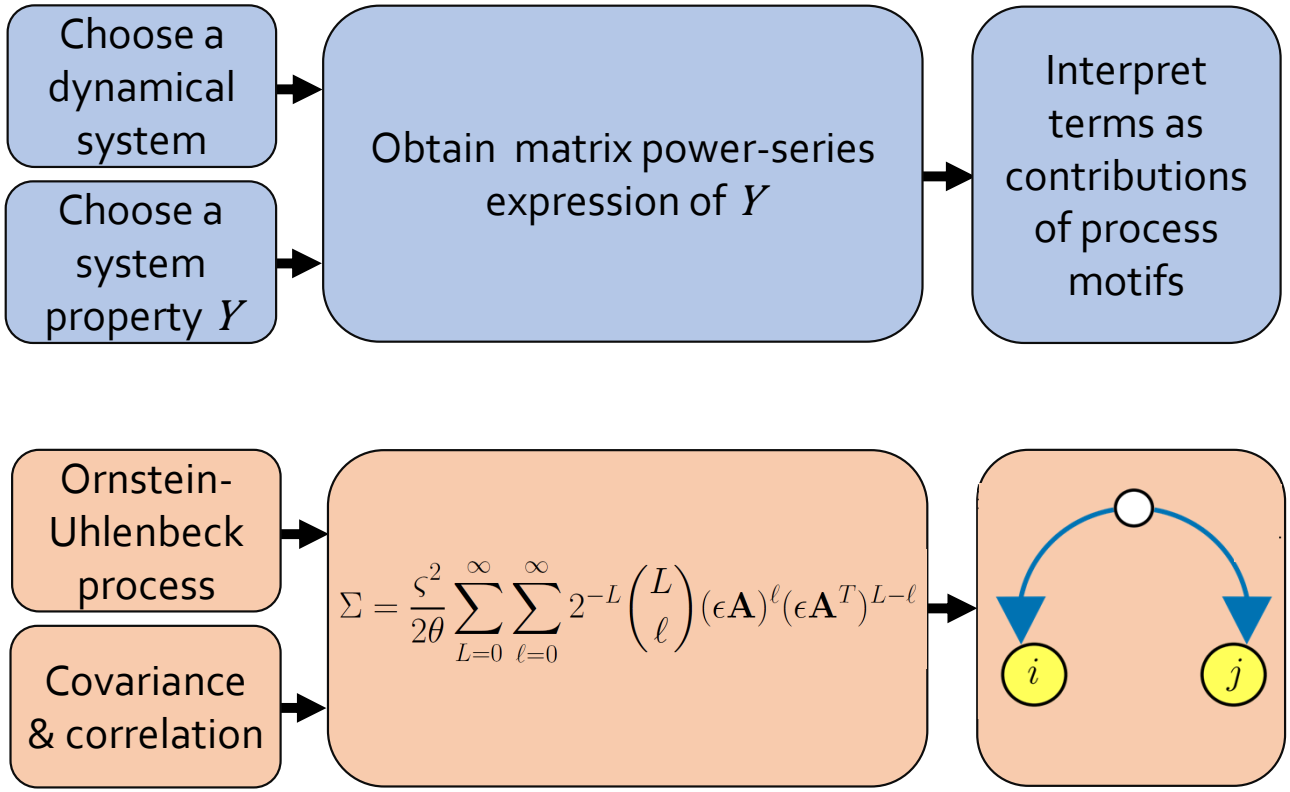
L
i
j

Matrix powers and walks in networks

Matrix	Edges	Walks
$\mathbf{A}^5$		
$(\mathbf{A}^T)^5$		
$\mathbf{A}^3(\mathbf{A}^T)^2$		

Pipeline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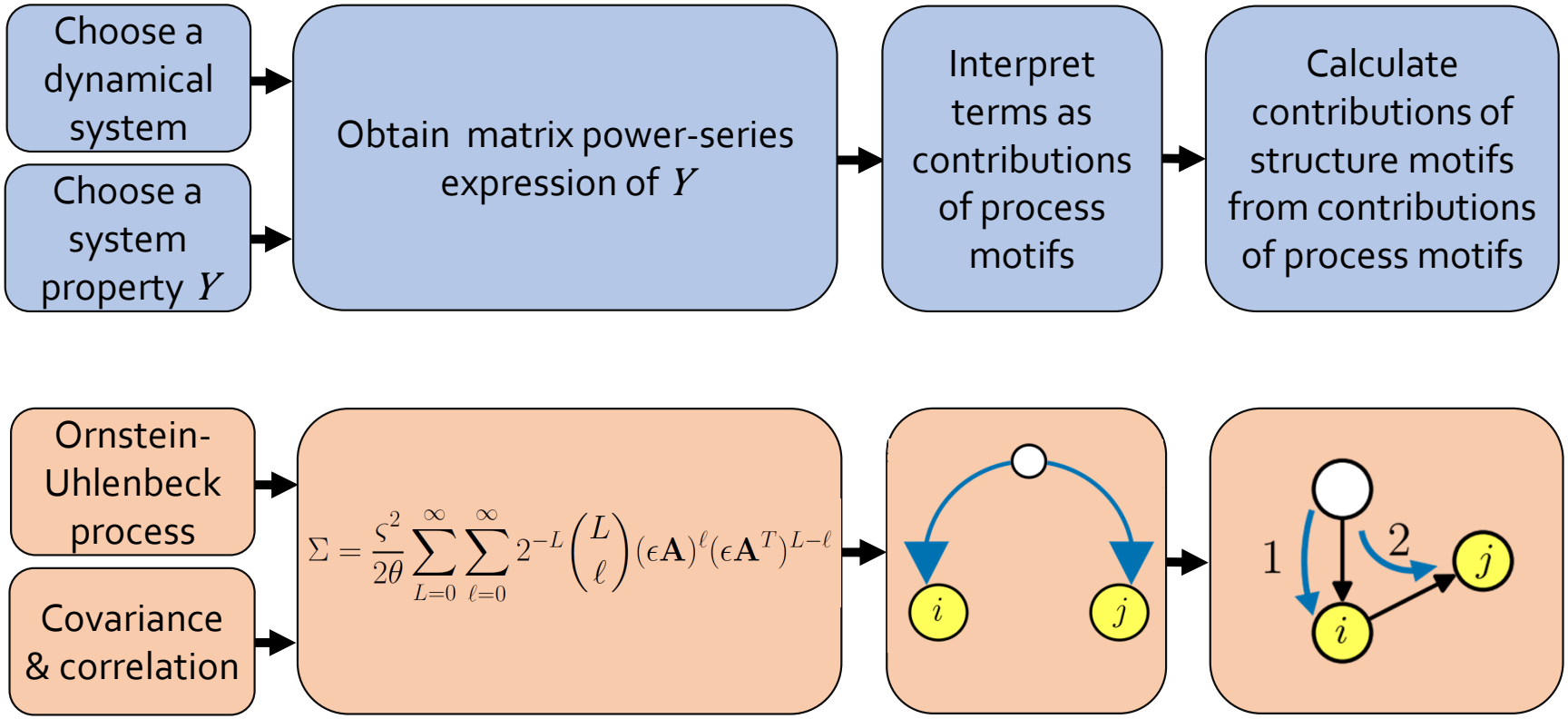
i
j

Pipeline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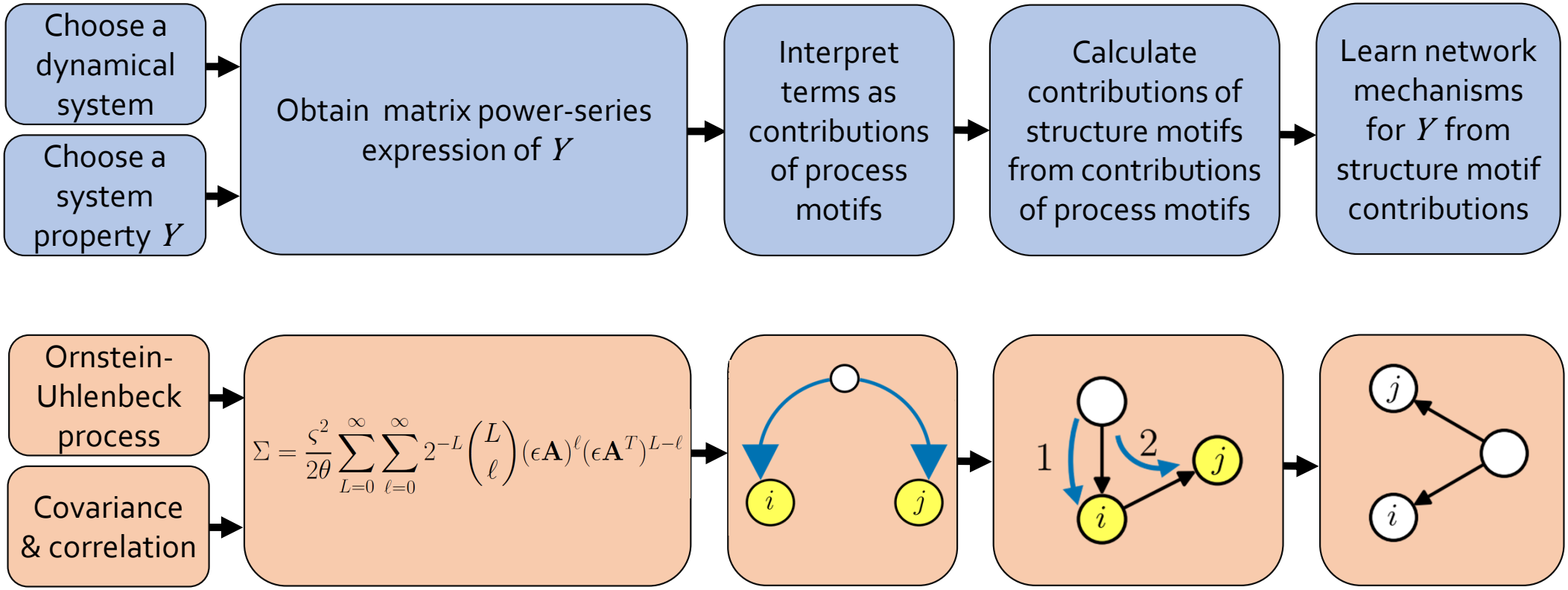
i
j
1
2
i
j

Pipeline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Interpret terms as contributions of process motifs
Calculate contributions of structure motifs from contributions of process motifs
Learn network mechanisms for Y from structure motif contributions
Ornstein-Uhlenbeck process
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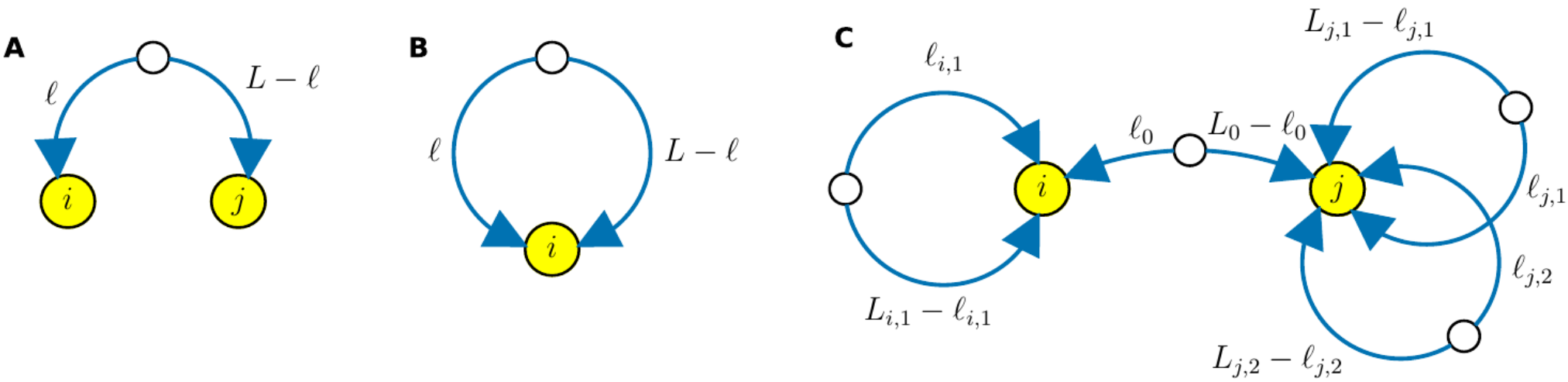
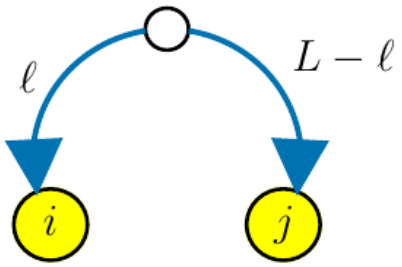
i
j
1
2
i
j
j
i

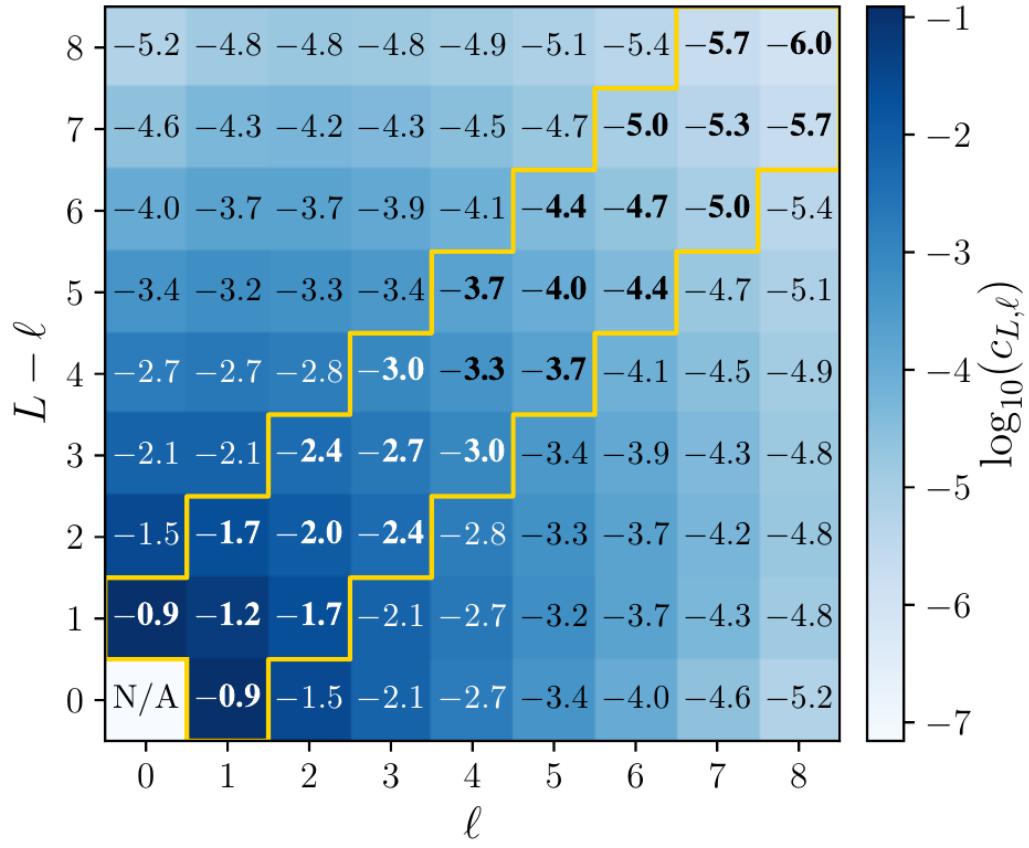
Process motifs



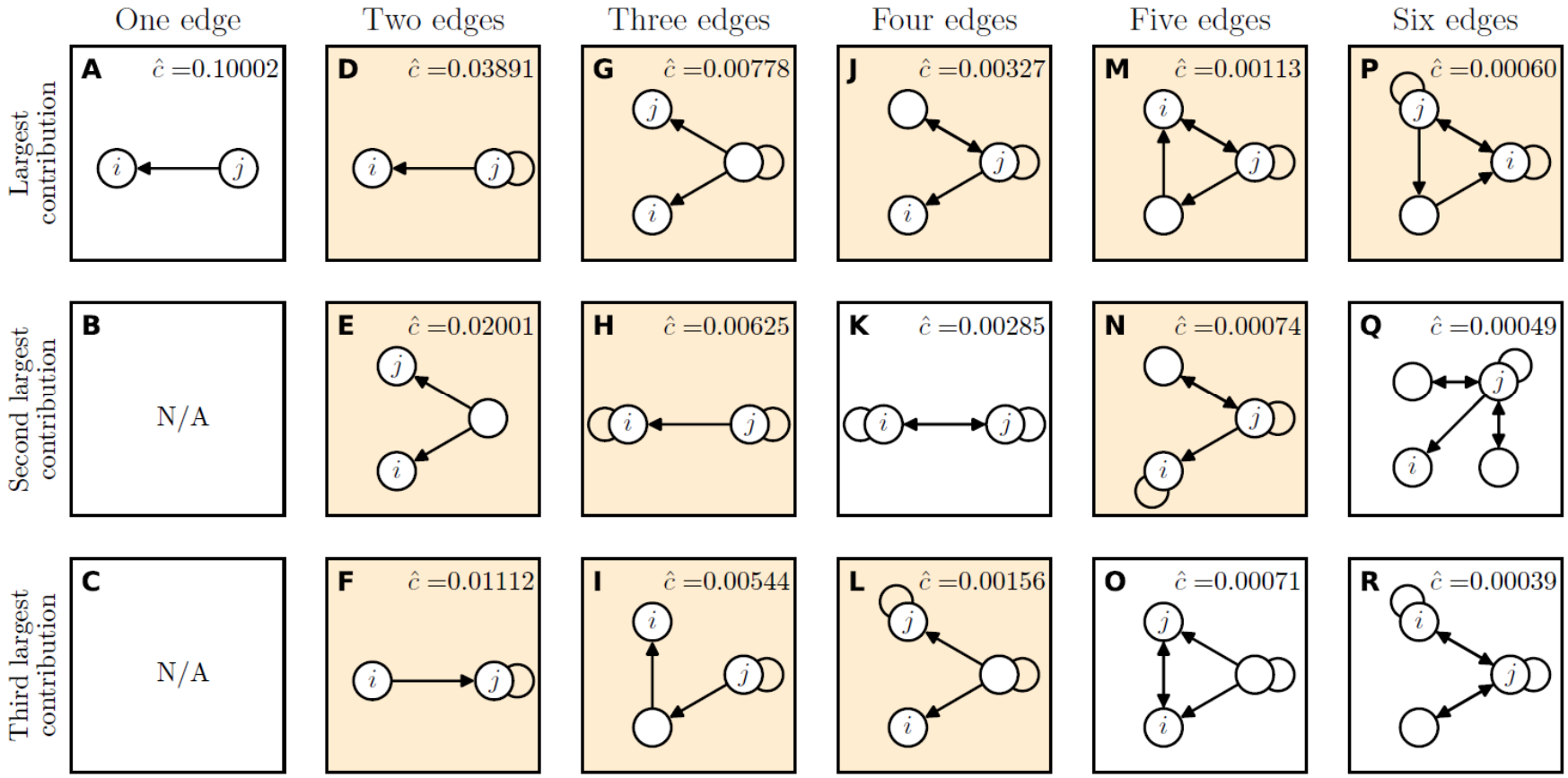
Process motifs for (A) covariance, (B) variance, and (C) correlation.

Contributions of process motifs

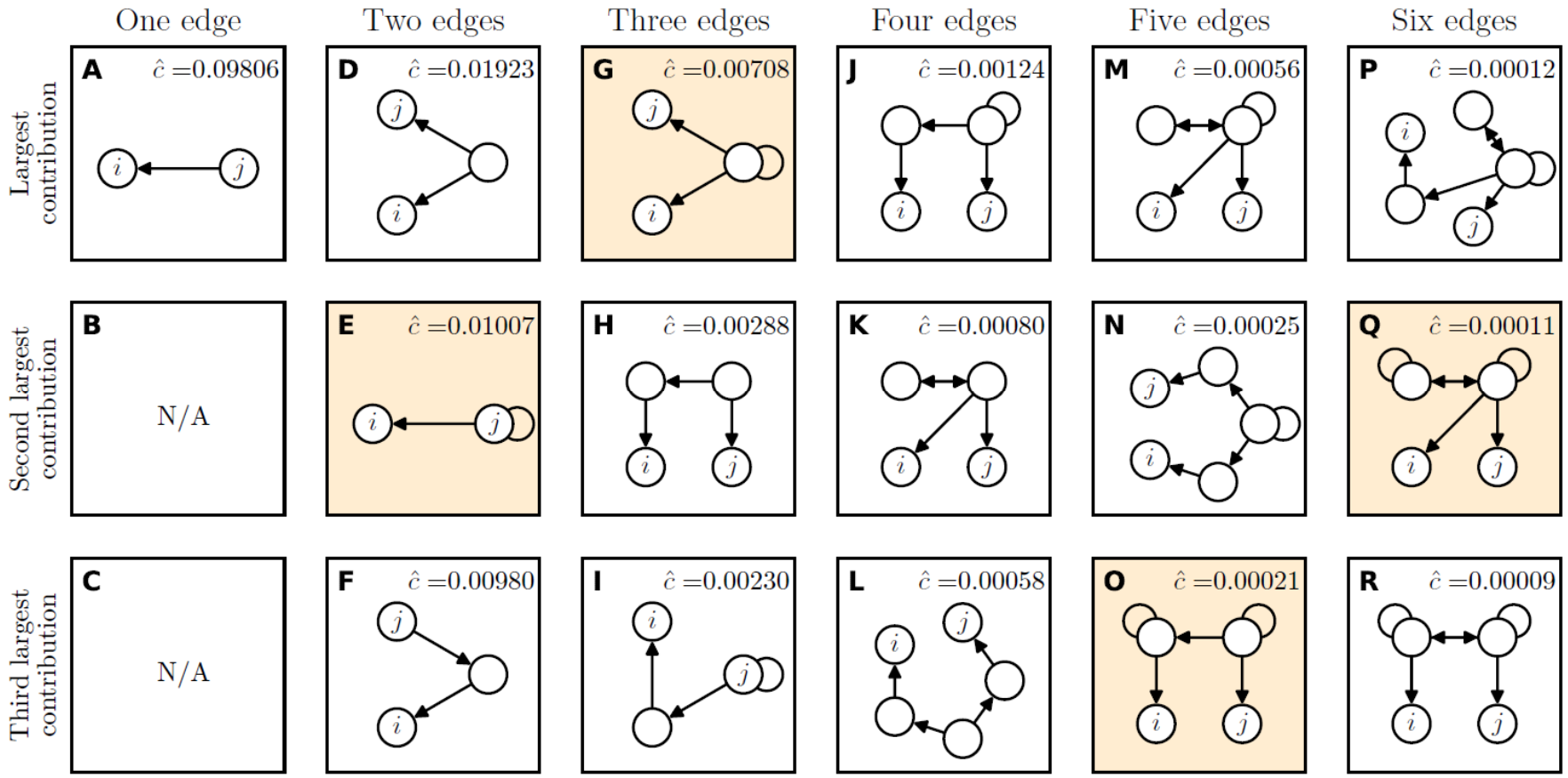




Contributions to covariance

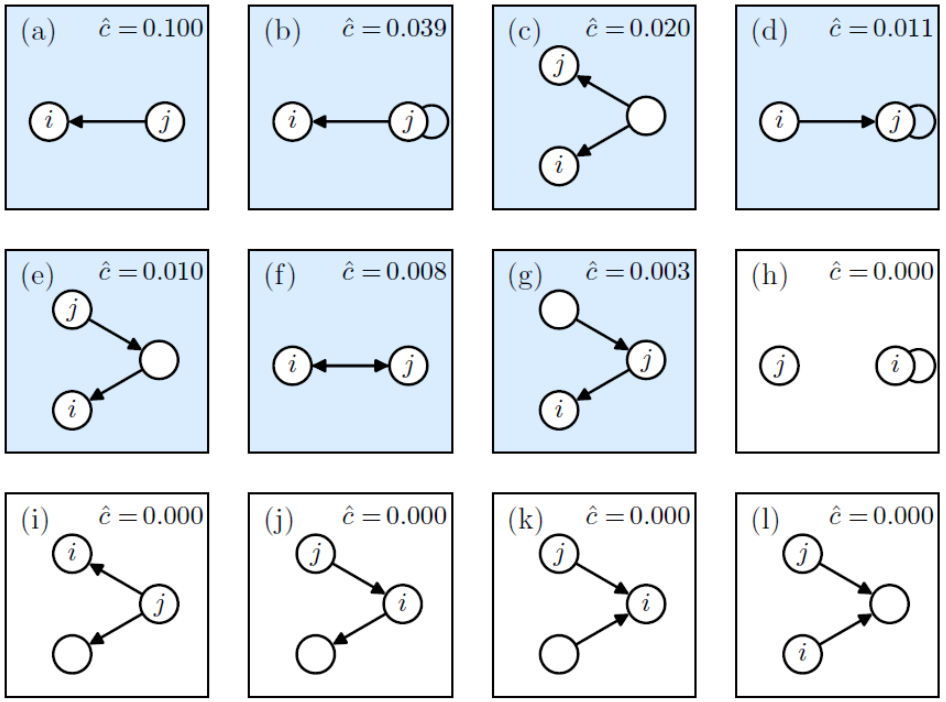


Contributions to correlation

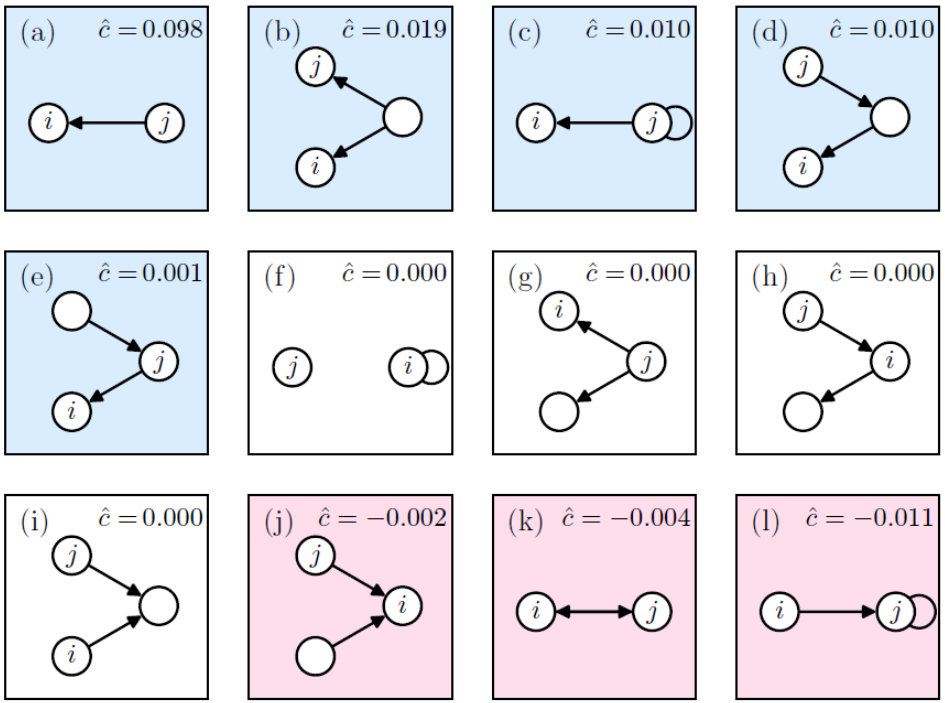


Emergence in 2-edge motifs

Covariance



Correlation



Motifs for processes: summary

- **Mechanistic connections** between structural motifs and their contribution to emergent properties of processes on networks
- Tool to explore the **role of recurrence** for dynamics on networks

Motifs for processes: summary

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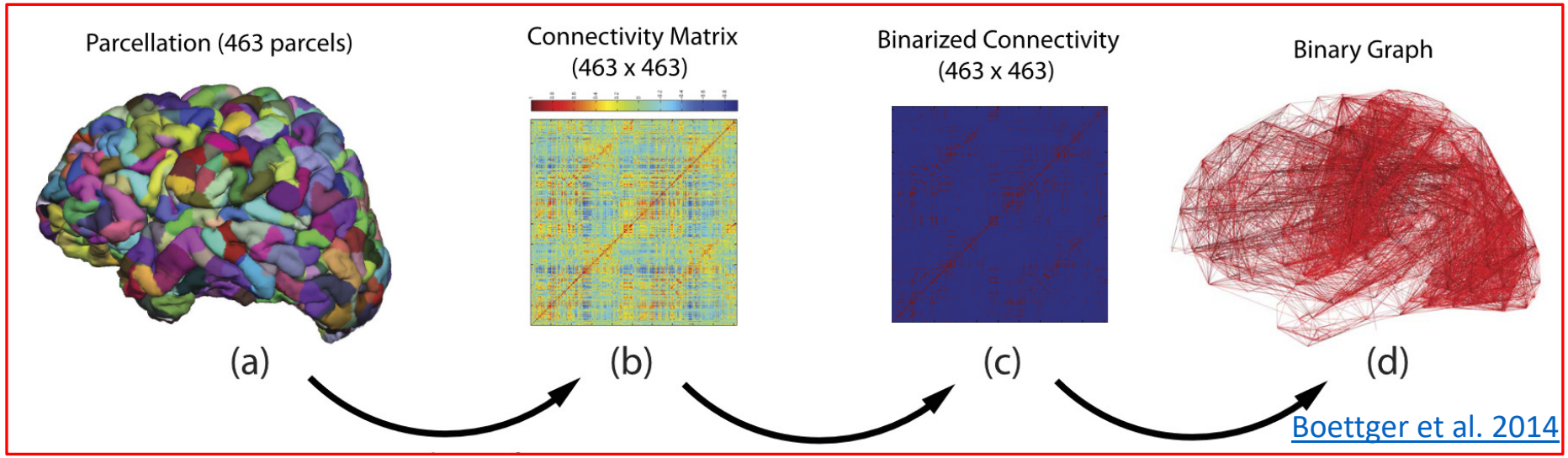

Boettger et al. 2014

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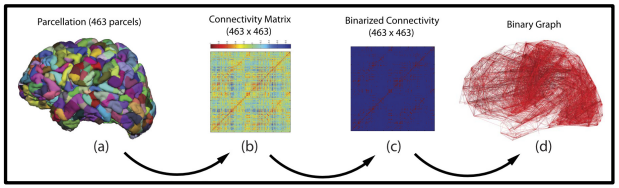

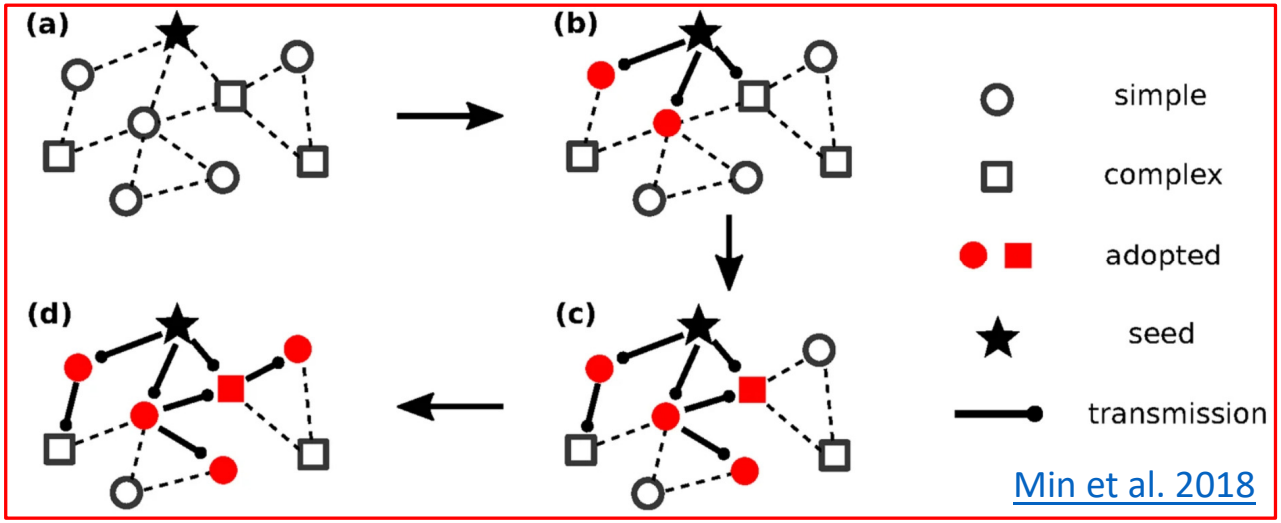

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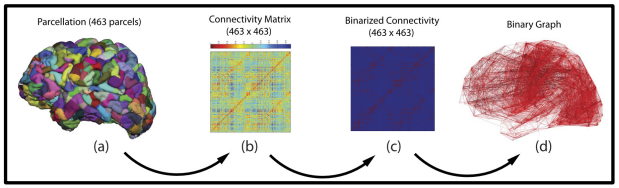

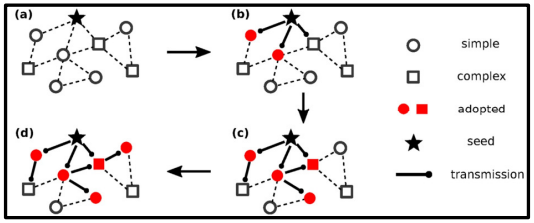

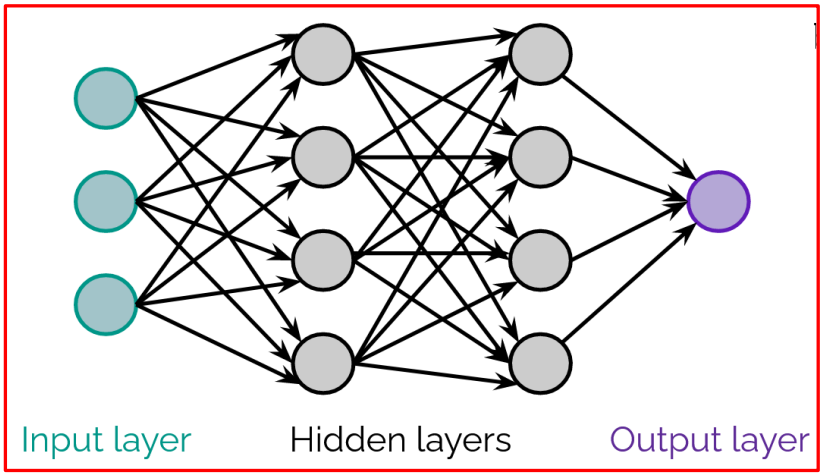

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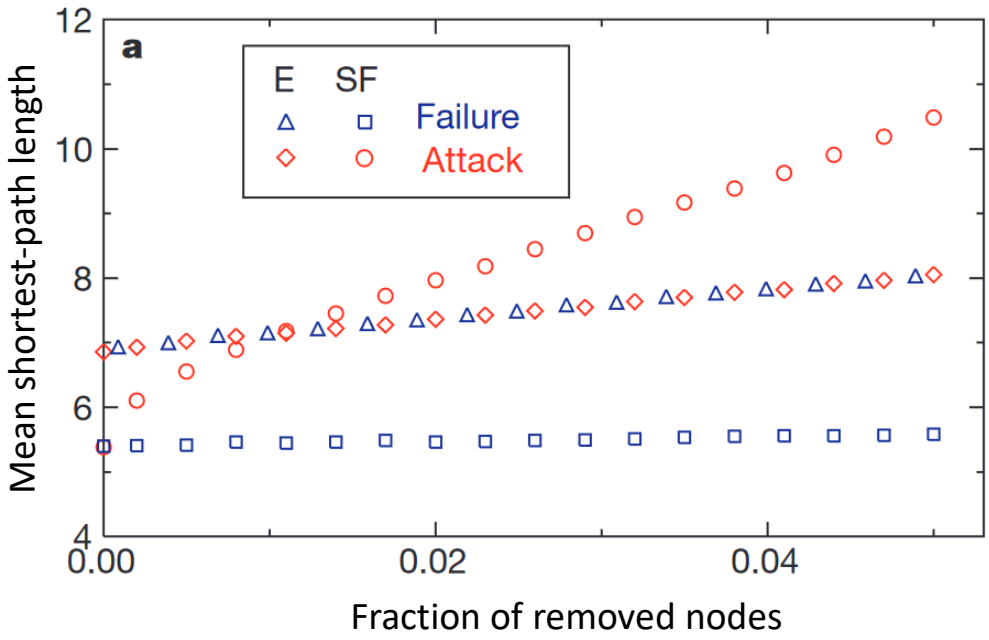
Network robustness and system design

Is a system robust?

Robustness



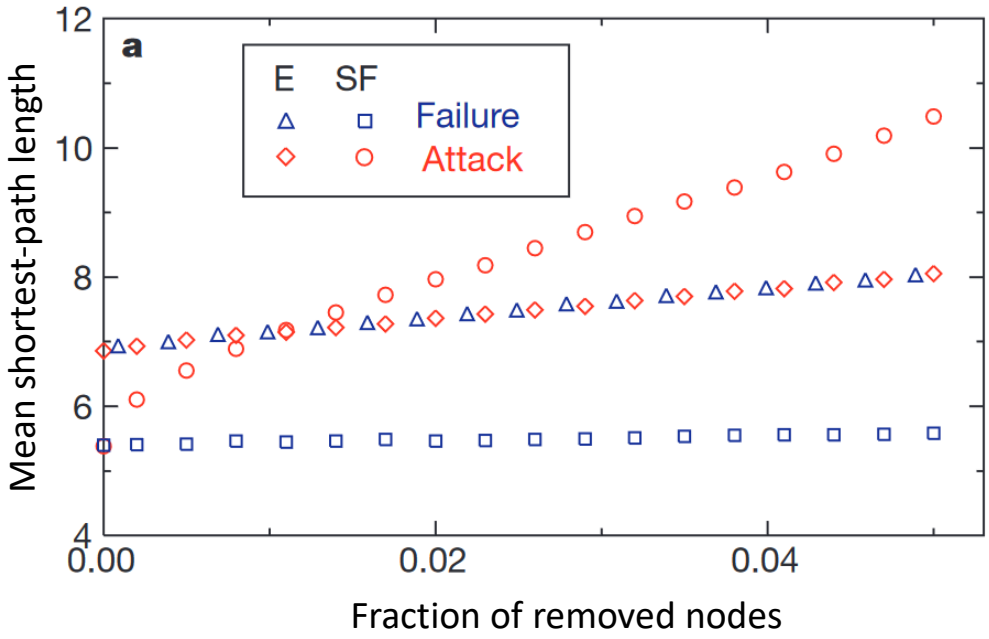
Albert et al. 2000

Is a system robust?

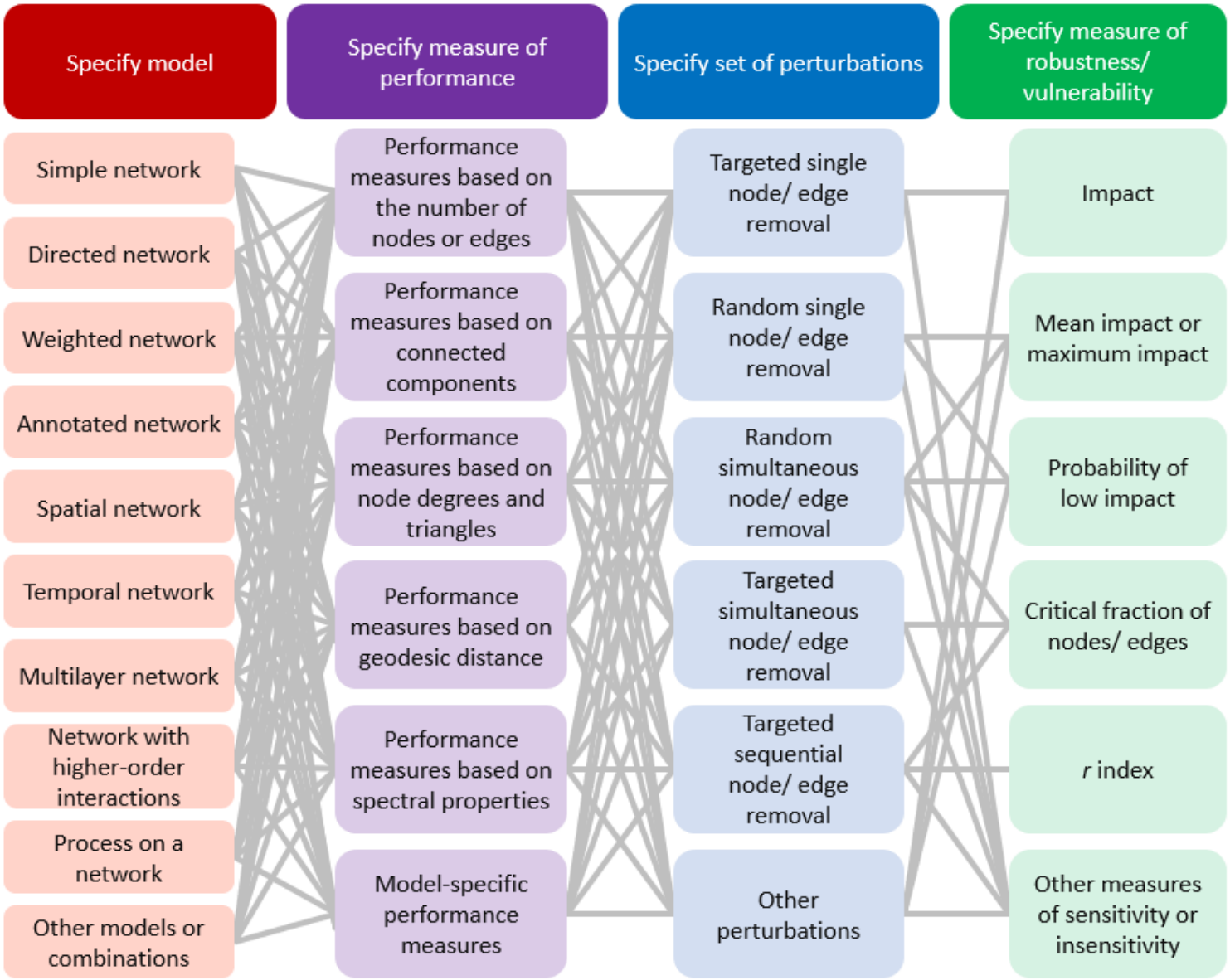
Robustness

Insensitivity of a **function**
of a **model** to a **perturbation**

Doyle et al. 2010



Albert et al. 2000


Specify model
Specify measure of performance
Specify set of perturbations
Specify measure of robustness/ vulnerability
Simple network
Directed network
Weighted network
Annotated network
Spatial network
Temporal network
Multilayer network
Network with higher-order interactions
Process on a network
Other models or combinations
Performance measures based on the number of nodes or edges
Performance measures based on connected components
Performance measures based on node degrees and triangles
Performance measures based on geodesic distance
Performance measures based on spectral properties
Model-specific performance measures
Targeted single node/ edge removal
Random single node/ edge removal
Random simultaneous node/ edge removal
Targeted simultaneous node/ edge removal
Targeted sequential node/ edge removal
Other perturbations
Impact
Mean impact or maximum impact
Probability of low impact
Critical fraction of nodes/ edges
r index
Other measures of sensitivity or insensitivity

Link between structure and dynamics

Centrality measures

Measures of performance

Link between structure and dynamics

Centrality measures

Betweenness centrality — shortest

Eigenvector centrality — all (weighted)

Degree centrality — Length = 1

Measures of performance

Link between structure and dynamics

Centrality measures

Betweenness centrality — shortest

Eigenvector centrality — all (weighted)

Degree centrality — Length = 1

Measures of performance

Link between structure and dynamics

Centrality measures

Betweenness centrality — shortest

Eigenvector centrality — all (weighted)

Degree centrality — Length = 1

Measures of performance

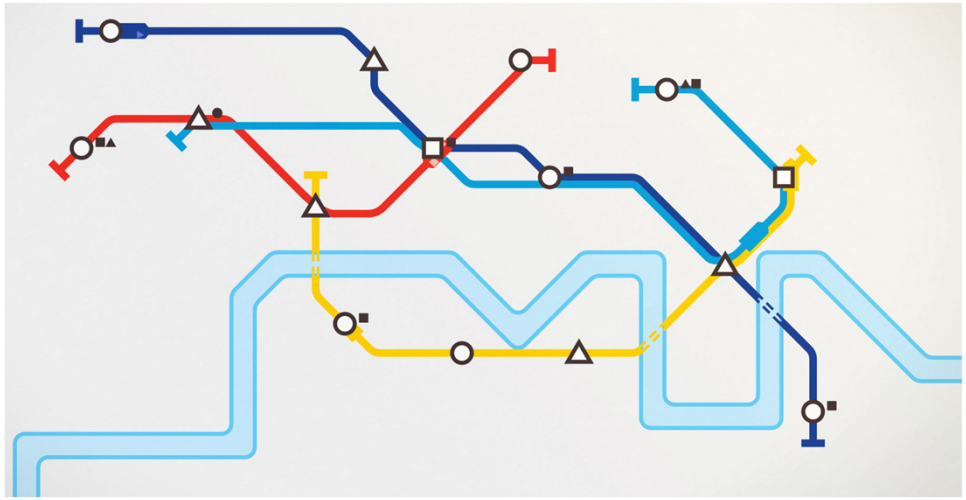
Mean shortest-path length (Efficiency) — shortest

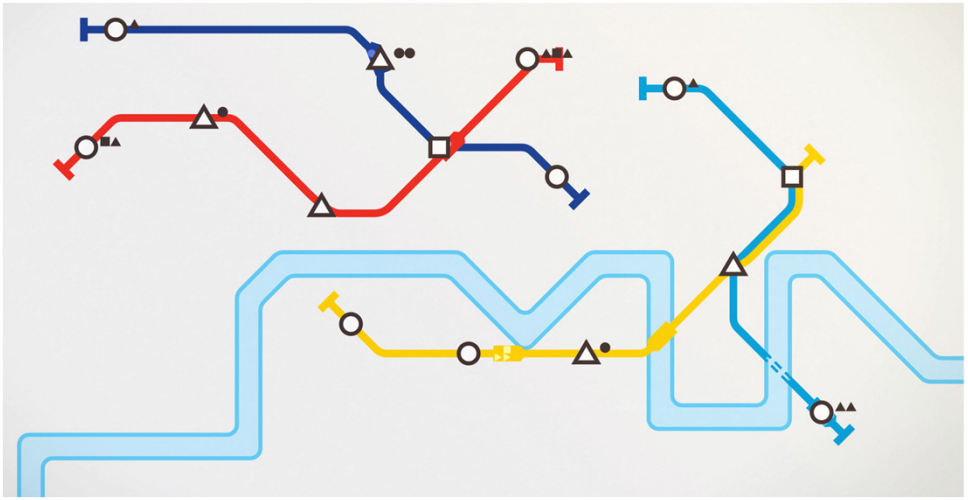
Size of the largest connected component — any

Communicability — all ST (weighted)

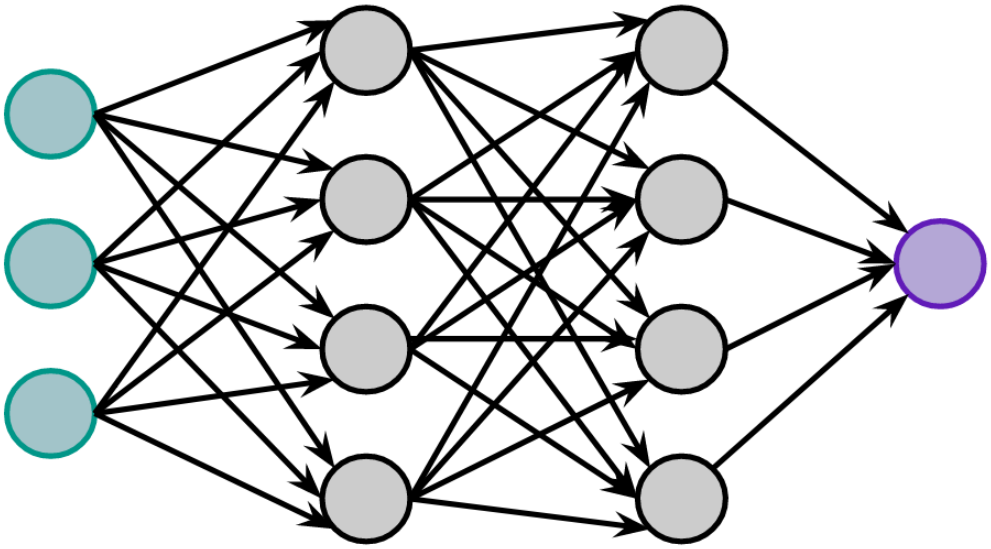
Short or long walks, short or long paths

- View performance measures as functions of walks or paths
- Identify good performance measures from important walks/paths





System design in machine learning

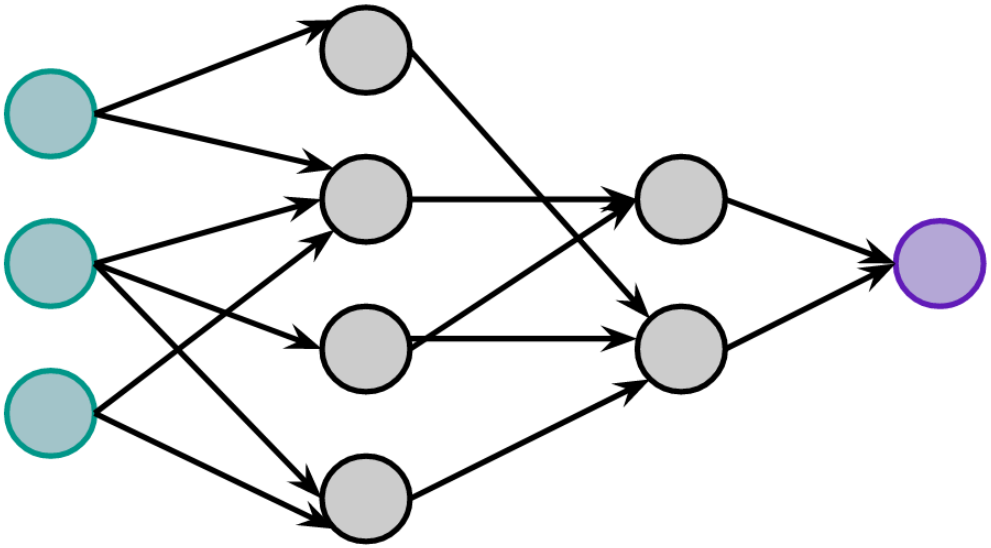


Dense, redundant, robust network

Input layer Hidden layers Output layer

System design in machine learning

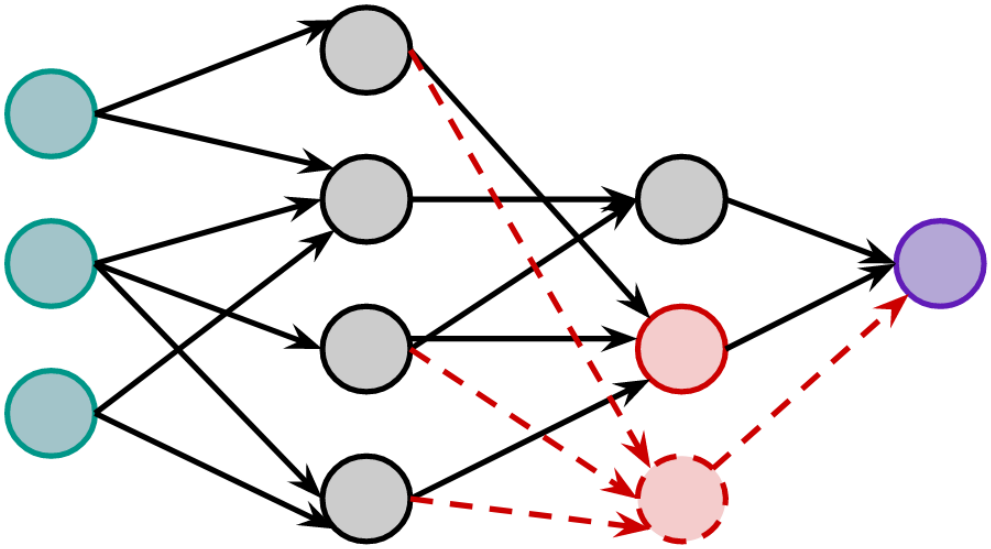
Sparse, non-redundant,
non-robust network



Input layer Hidden layers Output layer

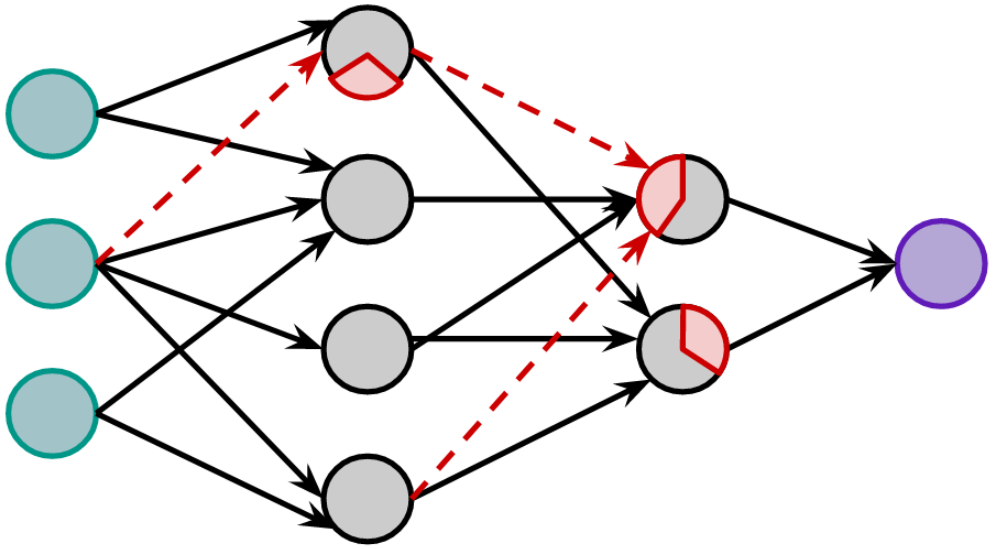
System design in machine learning

Increase robustness via increasing redundancy



Input layer Hidden layers Output layer

System design in machine learning



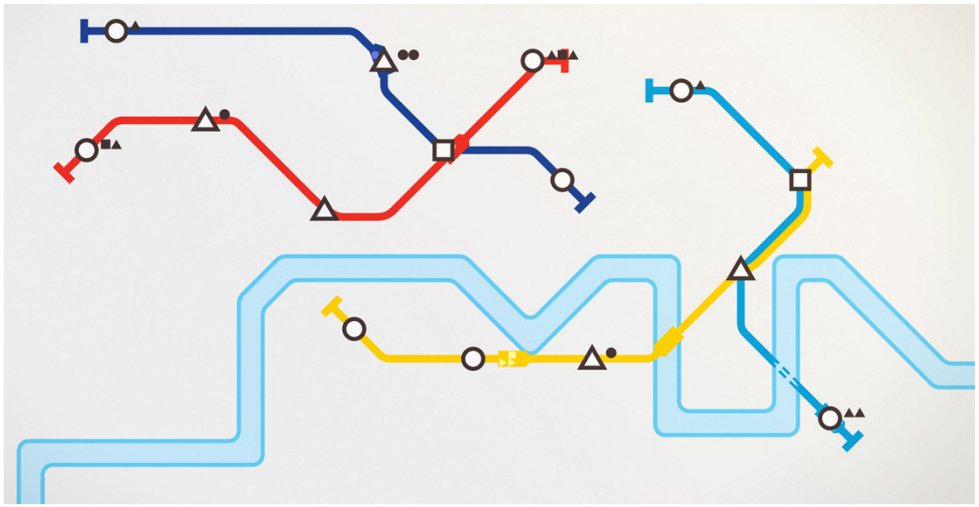
Input layer Hidden layers Output layer

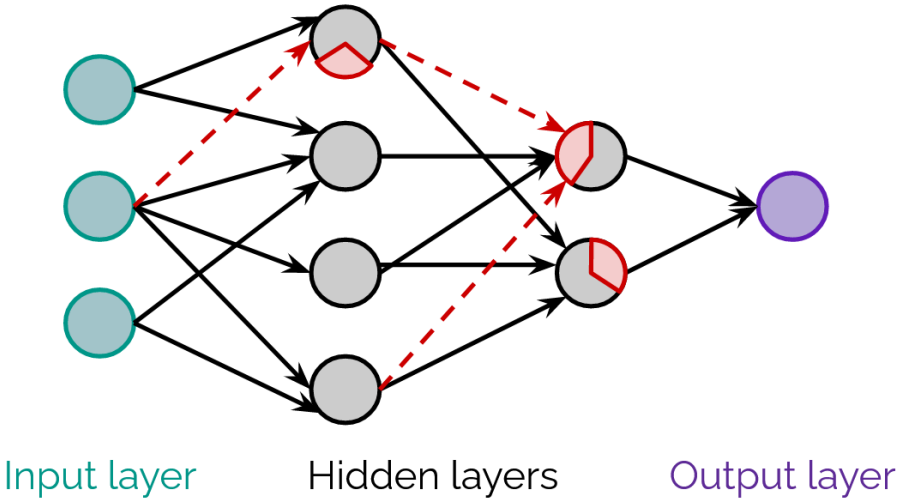
Increasing robustness via
increasing degeneracy

System design: summary

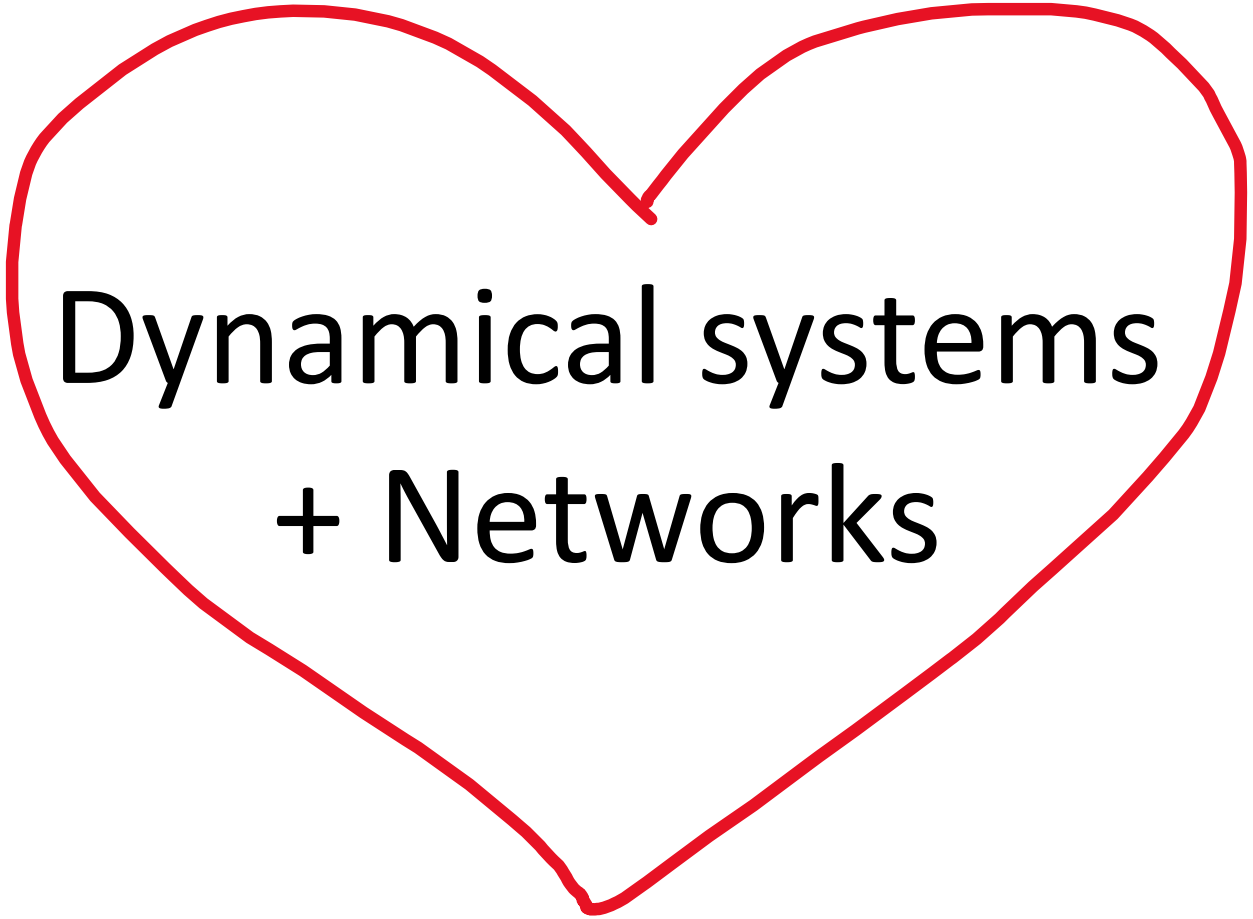
Microprocesses (walks, paths) help with

- the choice of a performance measure for studies of robustness
- strategies for designing sparse, robust networks.

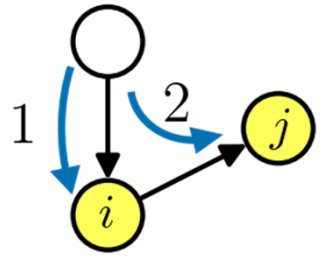




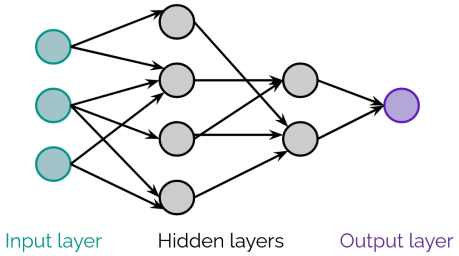
Conclusions



Conclusions

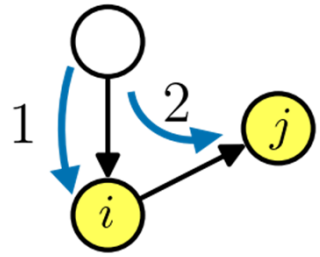


Motifs for processes on networks

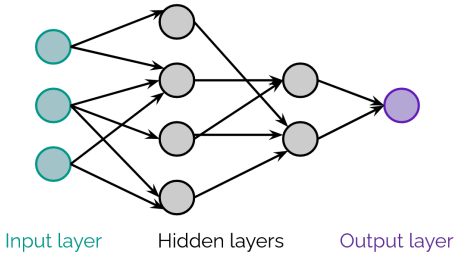


Network robustness and system design

Conclusions

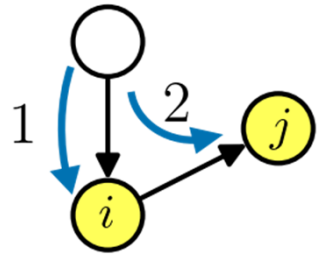


Motifs for **processes**
on networks



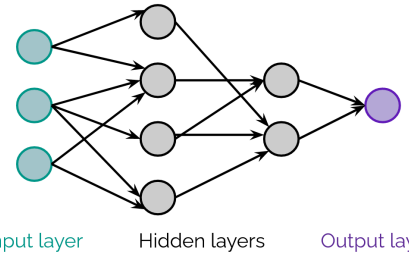
Network **robustness**
and **system design**

Conclusions



Motifs for **processes** on networks

Mechanisms



Network **robustness** and **system design**

Design and optimization

Plug #1: Myself!

Mathematical modelling, dynamical systems, complex systems, networks in biology and ecology, natural and artificial neural networks, system robustness and system redundancy

On the market for new adventures starting in 2021!

Plug #2: Virtual WiNS seminar!

All welcome!

New seminar talks starting in October!