Motifs for processes on networks

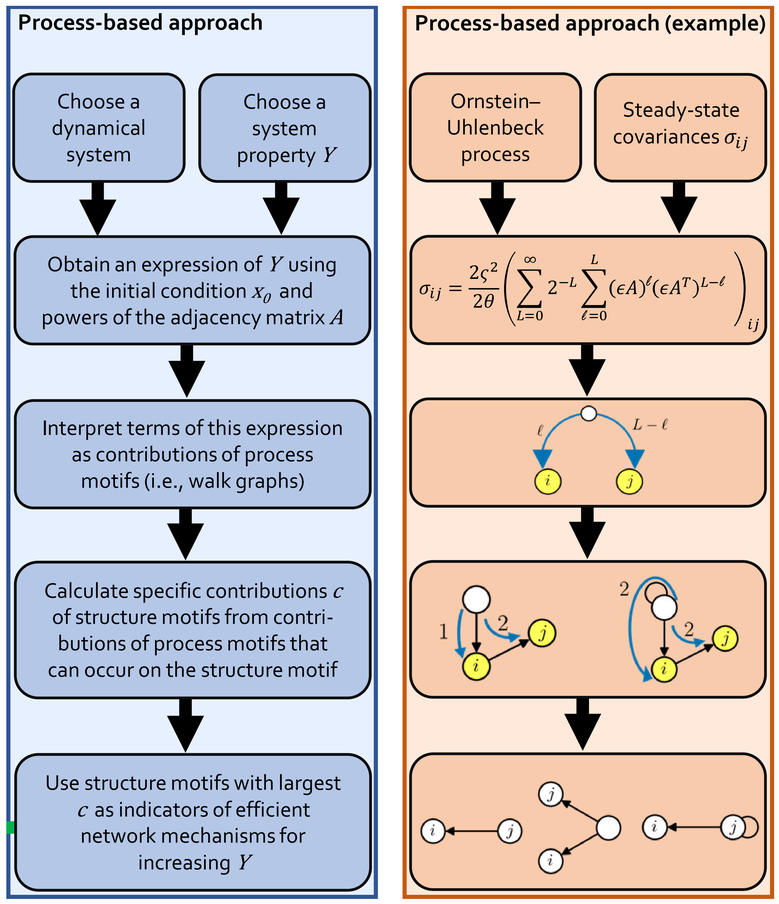
Alice C. Schwarze*, Mason A. Porter†

*Department of Biology, University of Washington, Seattle, WA, USA
†Department of Mathematics, University of California Los Angeles, Los Angeles, CA, USA

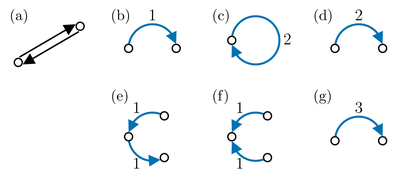
DOWNLOAD: https://tinyurl.com/motif-poster

DEMO: https://tinyurl.com/motif-demo

Overview

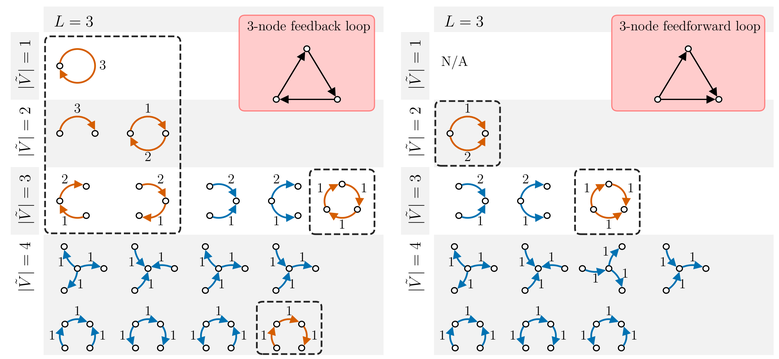


What are process motifs?



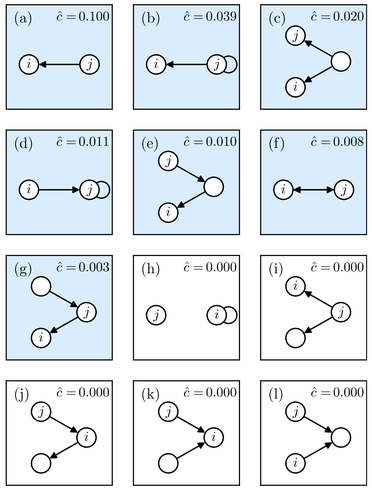
We define a process motif to be a structured set of walks on a network. For the small network in (a), panels (b)–(g) show examples of walk graphs that can occur on it.

Structure constrains processes on networks

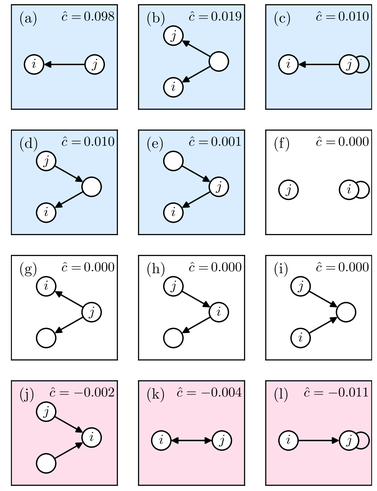


Structure motifs and specific contributions $\widehat{c}$

Contributions $\widehat{c}$ to covariance



Contributions $\widehat{c}$ to correlation



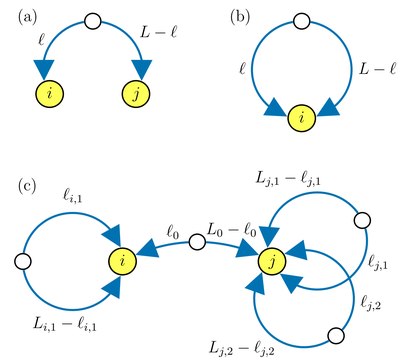
The Ornstein–Uhlenbeck (OU) process

One can describe the multivariate Ornstein–Uhlenbeck process via

$$d\mathbf{x}_{t+dt} = \theta(\epsilon\mathbf{A} - \mathbf{I})\mathbf{x}_t \, dt + \varsigma \, dW_t \, ,$$

where $\mathbf{x}_t \in \mathbb{R}^N$ is the state of the process and $\mathbf{A}$ a directed and/or weighted adjacency matrix. Parameters include the *reversion rate* $\theta > 0$, the *noise amplitude* ς^2, and the *coupling parameter* $\epsilon > 0$.

Process motifs for the OU process



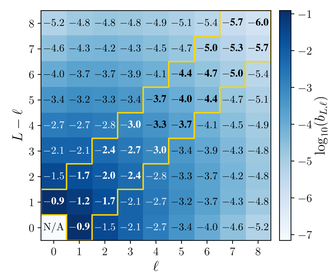
Process motifs for (a) covariance, (b) variance, and (c) correlation.

Process motifs for correlation consists of a process motif for covariance and a non-negative number of process motifs for variance. Every process motif for covariance is thus also a process motif for correlation. However, not every process motif for correlation is a process motif for covariance.

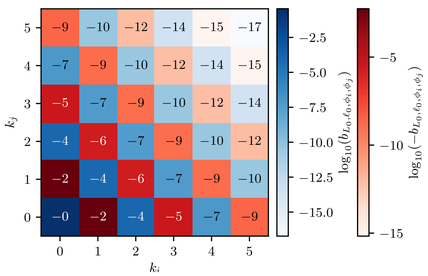
Process-motif contributions

Process motifs for covariance depend on the parameters L and ℓ. Process motifs for correlation depend on the parameters L and ℓ and the parameter sets ϕ_i and ϕ_j (with $|\phi_i| = k_i$ and $|\phi_j| = k_j$). The contributions of process motifs to covariance and correlation depend on these parameters.

Contributions b to covariance



Contributions b to correlation



Contributions of motifs to system functions

It is sometimes possible to write a function Y of a state $\mathbf{x}$ of a dynamical system as

$$Y = \sum_i b_{p_i} n_{p_i} \, ,$$

which is a weighted sum of counts of n_{p_i} process motifs p_i. The real value b_{p_i} is the *contribution* of the process motif p_i to Y. The *specific contribution* $\widehat{c}$ of s is the sum of contributions of process motifs that can occur on s but not on subgraphs of s. One can write Y as weighted sum of counts n_{s_i} of structure motifs.

$$Y = \sum_i \widehat{c}_{s_i} n_{s_i}$$